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Comparative Analysis of Physics-Informed Neural Networks Using TOPSIS: Balancing Efficiency, Interpretability, and Physical Law Integration

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Abstract: *This study evaluates the performance of Physics-Informed Neural Networks (PINNs) and related methods in scientific computing and engineering applications using the Technique of Selection by Ideal Solution (TOPSIS) method. The analysis compares traditional numerical methods, machine learning models, hybrid physical-ML models, Gaussian process regression, and index regression on four key criteria: data efficiency, interpretability, computational costs, and convergence of physical laws. The results reveal that indexed regression emerges as the best ranking approach, demonstrating well-balanced performance across all criteria. It excels in data efficiency and interpretation while keeping reasonable computational costs and adhering to the laws of physics. Gaussian process regression is second, particularly robust in interpretation, making it suitable for applications requiring transparent decision processes. Hybrid physics-ML models show strong performance, especially in the integration of data efficiency and physics laws, effectively combining traditional physics-based modeling with machine learning capabilities. Traditional numerical methods, while valuable in some contexts, show limitations in data efficiency and interpretation compared to newer approaches. Pure machine learning models rank very low, suggesting that data-driven approaches may not be sufficient for problems where physical laws play an important role. This study underscores the importance of balancing multiple criteria when selecting modeling approaches for physics-informed problems. This highlights the trend towards hybrid methods that combine domain knowledge with data-driven learning, addressing the limitations of purely data-driven or physics-based approaches. These findings provide valuable insights to researchers and practitioners in scientific prediction and engineering, future modeling of complex physical systems, and approaches that effectively combine field knowledge with data-driven techniques to optimize complex physical phenomena.*

Keywords: *Physics-Informed Neural Networks (PINNs), TOPSIS Method, Symbolic Regression, Gaussian Process, Regression, Hybrid Physics-ML Models, Data Efficiency, Interpretability, Computational Costs, Scientific Computing and Machine Learning.*

1. INTRODUCTION

Physics-informed neural networks improve automatic differentiation by eliminating the need for mesh generation. However, PINNs are not designed to replace traditional computational fluid dynamics (CFD) codes. Currently, PINNs are less accurate and efficient than higher-order CFD codes for static forward problems. This limitation stems from the loss minimization function, which is a high-dimensional, non-convex function, presenting challenges in commercial machine learning applications. For the flow problem in question, any CFD solution outperforms PINNs in accuracy and efficiency. In addition, since the forward and reverse formulas of PINNs are the same, expensive data is not required, especially for historical design updates and flow issues that prevent progress in some applications [1]. Deep learning is particularly useful in areas such as computer vision, natural language processing, and game theory, revolutionizing how we classify applications in various fields, especially pattern recognition and regression tasks. It distanced itself from traditional applied mathematics such as solving partial differential equations. Deep neural networks are increasingly used to deal with complex problems, especially those with strong nonlinear relationships, convective dominance, or shocks challenges that traditional numerical methods struggle to address. This innovative approach to scientific computing has shown great promise; For example, in neuroscience, it helps global approximation of neural networks. Additionally, it serves as an efficient method for building meta-models that can rapidly predict dynamic systems. Research indicates that neural networks can effectively capture essential nonlinear input-output relationships in complex systems [2]. The methods outlined mainly use supervised learning, requiring a labeled dataset (standard truth) for model training. The primary benefit of these data-driven methods is the fast post-training prediction speed. However, generating sufficient data can be very challenging, and the ability of the model to generalize across different experimental conditions is not guaranteed. Additionally, other techniques have been explored, but they generally do not address heat transfer problems because they do not consider the underlying physics. To overcome this, a multitask learning approach is required, such as physics-informed neural networks, which combine data with partial differential equations to solve forward and inverse problems. Initially designed for this purpose, PINNs have been used in a variety of fluid dynamics and heat transfer situations [3]. Partial differential equations are frequently used in scientific fields to model various engineering phenomena, usually stemming from fundamental principles such as conservation of mass or energy. However, it is often impractical to analyze these PDEs to obtain solutions for many real-world systems. Consequently, countless numerical methods such as the finite element method and pseudo-spectral methods have been developed to approximate their solutions and behavior. However, many of these systems are too complex for straightforward turbulence simulations, and numerical integration techniques can be resource-intensive, often requiring significant computational power for convergence [4]. Fields such as image recognition, robotics, and weather forecasting see significant benefits from these methods. In energy systems, decision trees and neural networks can address dynamic and optimization challenges, reaching solutions three times faster than traditional computational methods. However, applications of machine learning in power systems and other physical systems often overlook the underlying physical models. This oversight has introduced significant biases in the quality of training data, requiring large datasets and often complex neural network architectures. Although recent advances indicate the potential for efficient dataset generation, the training process demands substantial computational resources to further expand the size of the dataset [5]. Developing efficient and accurate numerical methods to simulate the complex physical and multidimensional dynamics of biomedical phenomena is an ongoing challenge in scientific computing. This section focuses on stationary and convex equations (PDEs) designed to provide accurate approximations. These techniques usually involve transforming the computational domain into a grid based on a mathematical model, adjusting reciprocal values and smoothing the results, or constructing linear systems of PDEs at the grid nodes. The unknown state variables or their functional expansions are then solved to find the coefficients. Notably, spectral methods treat the model or nodal approximation as a linear combination of known basis functions, which creates a unique solution space for the problem within a finite framework. Using an equation with these functions, the resulting system is checked for correctness [6]. In addition, researchers such as De Rigg et al. have applied PINNs to various nonlinear PDEs, providing the first comprehensive theoretical analysis of XPINNs. Penalizing gradients at interfaces greatly improves the performance of XPINNs, especially in inverse shock wave problems within high-velocity compressible flows, governed by incompressible Euler's equations. Although these equations account for perturbations or shocks, the states remain stable, making it challenging or impossible to handle such inverse problems with traditional numerical methods [7]. In traditional numerical methods, inverse problems and inverse design are tackled using similar techniques because the PDE can be solved directly by numerical solutions. This approach involves minimizing an objective function to identify an optimal PDE solution consistent with the observed data. However, with PINNs, reverse design presents additional challenges. In this context, the difference between the PDE solution and the data is considered as the loss function (data-based loss). PINNs solve inverse problems by combining this data-based loss, while aiming to reduce PDE-based loss. This dual focus enables efficient training of the network. Since these two types of losses are aligned and both losses are reduced to zero simultaneously, solving the resulting optimization problem is relatively straightforward [8]. Despite recent progress, many studies have primarily focused on solving straightforward forward or inverse problems within simple geometric domains, which limits the applicability of PINNs to more complex real-world scenarios. Various strategies have been proposed to deal with the problem of input domains in neural networks. One approach involves using a signed distance function to describe the domain boundary, enabling the implementation of boundary conditions through penalties in the loss function instead of time dependence. Another approach involves breaking the domain into smaller, simpler sub domains for easier modeling. In addition, convolution neural networks have been suggested to improve coordinate mapping between these simple sub domains. However, most of these studies present examples in two dimensions [9]. In recent years, autonomous vehicles, speech recognition, medical diagnostics, and consumer technologies have proliferated, with sophisticated systems increasingly leveraging machine learning techniques. However, the application of these methods in scientific calculations has received less attention. To address this gap, a new approach called physics-informed neural networks has emerged, which combines machine learning with scientific computation. Thanks to their straightforward nature, PINNs have made significant advances in various fields of computational science, demonstrating efficiency in solving forward problems, obeying physical laws, and solving inverse problems in environments with unknown physics or governing equations. Nevertheless, a major drawback of PINNs is their computational intensity, especially during network training, which is very demanding, especially for forward diversity problems [10]. Fully data-driven approaches based on deep learning are often unavailable for many scientific problems, mainly due to the need for large training datasets. Furthermore, these models often ignore physical constraints and do not effectively integrate observational data. Although they provide a good fit, they often fail to conform to basic physical principles. Therefore, incorporating the governing physics and domain knowledge into the modeling process is crucial to ensure accuracy and robustness in the results. In addition to observational data, using domain knowledge as valuable input before training a model improves its performance and deepens the understanding of the physical or mathematical aspects involved in the calculations [11]. The purpose of this manuscript is to demonstrate how advanced learning techniques can improve scientific engineering. In general, scientific computing methods yield clear predictions from interpretable models and can provide measurable error bounds. Conversely, machine learning can automatically adapt based on data, providing rapid predictions for new phenomena after training. Physics-informed neural networks exploit the strengths of both approaches. This manuscript will explore how adaptive numerical analysis techniques such as quadrature can be used with trained PINNs to ensure efficient analysis and fast predictions [12]. Fast Marching Method and Fast Sweeping Method are two commonly used techniques for solving the symbolic equation. FMM belongs to a class of algorithms known as single-pass methods. It was initially introduced by Jan Tsitsiklis, who used theoretical division to deal with the small equation, drawing on the shortest path algorithm developed by Dijkstra. Shortly thereafter, a finite-difference method was introduced that incorporated elements of Dijkstra's sorting and updating processes. FMM uses entropy-satisfying gradient approximation schemes to efficiently solve the regional equation in one pass through a fast sorting mechanism [13]. During training, this loss function is minimized, reflecting the difference in governing regions calculated directly from the relevant equations. It consists of residuals estimated at summation points, which can be weighted using the Gal Kin method or derived from the power function of the Euler–Lagrange differential equation. Instead, the input data correspond to points in the problem domain defined by the partial differential equations. This enables an approximate solution where the loss function can be adjusted based on the distance between the predicted and measured values. Additionally, one of the partial differential equations or its higher-order coefficients may be assumed to be unknown during training [14]. This adaptive activation function exhibits better learning capabilities compared to classical methods, especially at early training stages, significantly increasing the convergence rate leading to improved solution accuracy. To better understand the learning process, we will analyze how the different frequency bands present in the solution are captured by a neural network solver in the frequency domain. This method enables us to obtain approximate solutions for forward problems and to find the parameters in the governing equations for inverse problems. Our simulation results demonstrate that the proposed neural network approach is simple and effective, resulting in significant improvements in performance, robustness, and accuracy, especially for forward problems involving differential equations [15].

2. MATERIALS AND METHOD

Traditional Numerical Methods: In engineering and scientific applications, numerical methods are used for modeling and scientific computing in areas such as simulating airflow over airplanes, evaluating ocean currents, solving electromagnetic problems, and modeling shuttle tank separation. Numerical methods are used to approximate equations when exact solutions are not possible by algebraic methods. These methods produce a series of approximations that converge toward an exact solution to an equation or system of equations.

Machine Learning Models: Machine learning models are designed to detect patterns in datasets or draw conclusions without having previously encountered specific data.. Machine learning (ML) is a branch of artificial intelligence that focuses on making predictions or decisions based on data. The primary types of ML include supervised learning, unsupervised learning, and reinforcement learning.

Hybrid Physics-ML Models: Hybrid machine learning integrates empirical data, domain knowledge, and application contexts to create efficient, descriptive, and reliable ML solutions. The overall goal is to integrate these elements to create robust models. Combining the two models creates a hybrid model that can provide significant benefits, especially when using the SDLC model to help customers who are unsure of their system needs. Hybrid models, a term also used in dynamic modeling, combine data-driven approaches. Many modern hybrid approaches mix machine modeling with machine learning, and there are various methods to achieve this integration.
Gaussian Process Regression: This model is used in unsupervised learning as a soft clustering technique. Gaussian process regression (GPR) is used to make predictions by training data on k-nearest neighbors. It works effectively with small datasets and provides predictions with uncertainty estimates. Along with the covariance represented by the kernel means, the prior mean and prior integration must be specified.
Symbolic Regression: Symbolic regression (SR) is a type of machine learning based on the principles of symbolic programming. It uses techniques from various scientific disciplines to integrate processes and derive complete analytical equations from data. Indexed regression is another machine learning approach that aims to produce interpretable results. Unlike other methods, such as random forests or neural networks, which tend to be opaque, symbolic regression models construct data in a way that scientists understand and seek to create maps that elucidate underlying relationships.
Data Efficiency: Data-enhanced learning is a form of machine learning that can navigate complex domains without the need for large datasets. In contrast, traditional machine learning algorithms typically rely on detailed data to make logical decisions. Their performance is affected by the structure of the data, and their scalability is determined by the complexity of the task.
Interpretability: Interpretation involves understanding the conclusions drawn from a model, particularly regarding its predictive capabilities and the reasoning behind its results. Terms such as comprehensible, intelligible, and explainable are associated with the various facets of interpretation. The decision-making process of an AI model highlights the importance of understanding its workings. An explainable model offers transparency in its operations and clarifies the relationships between inputs and outputs. This approach enables clear and understandable explanations of the model's functionality.
Computational Costs: Credits are used when performing data loading and other Data Manipulation Language (DML) operations in the context of virtual warehouse computing for queries. Users manage virtual warehouses, allowing them to have direct control over credit consumption. Computational cost is a one-time charge incurred during simulation, specifically reflecting the execution time of each step. To measure the time required to run your model on real-time hardware, consider the simulation process for your target machine to estimate the processing time budget.
Incorporation of Physical Laws: A physical law, scientific law, or law of nature is derived from empirical observations of physical phenomena. Basically, it represents a scientific generalization. These laws are usually established through years of repeated observations and based on scientific experiments. The results of these experiments are universally recognized within the scientific community.

TOPSIS Method: By evaluating all possible alternatives in a decision problem, one can determine the best option among them. This paper examines the complexity effects associated with multi-criteria and multi-attribute models of alternative selection, with a particular emphasis on the Order Preference by Similarity to the Ideal Solution (TOPSIS) technique as a preference method. In real-world situations, data (attributes) may be imperfect or incomplete, which often leads to uncertainty because the data may be ambiguous or imprecise. In this study, the evaluation of each alternative and the weights assigned to each criterion are represented using trigonometric fuzzy numbers [16]. The TOPSIS method aims to find the optimal solution by evaluating the proximity of alternatives to this optimal solution while maximizing the available options. This approach ranks the alternatives, with the best option receiving the highest rank and the worst being assigned a rank of zero. Each alternative is positioned in this ranking system, creating a more nuanced assessment. The criteria used for selection should facilitate accurate comparisons between alternatives, as improper weights may lead to suboptimal effects and should be approached carefully. The steps of the TOPSIS technique are outlined, creating a geometric system in n-dimensional space to analyze a multi-criteria decision-making problem with m alternatives [17]. In today's economy, productivity levels are vital to generating revenue, making machine selection a critical decision for businesses looking to succeed in a competitive global marketplace. To develop an effective production system, companies need to identify the most suitable machines. However, the complexity of engine selection has increased due to the growing number of options and conflicting criteria. This research provides a decision support system to streamline the machine evaluation process. The proposed method includes two main steps: first, analyzing and identifying problem criteria and determining their weights for relevant sectors and sub-sectors using AHP; and second, ranking eligible alternatives using TOPSIS. This research provides a practical application of these methods to address real-world machine selection challenges [18]. In this model, subjective and objective criteria are classified. Fuzzy analysis uses a hierarchical approach and uses entropy weights to objectively estimate subjective weights. In addition, the combined weights are refined by integrating the results from both methods. Performance values for subjective criteria are expressed linguistically, while those for objective criteria are derived from objective evaluations. Standardized average integration method and modified distance method are used in integrated fuzzy TOPSIS approach. To demonstrate the computational process of this fuzzy TOPSIS algorithm, a hypothetical example involving the selection of a shipping company partner is presented [19]. The literature emphasizes the use of fuzzy logic methods to prioritize shopping websites and address uncertainty in the evaluation process. The main goal of this paper is to establish criteria for managing accuracy and determine the priority of measurement dimensions using a fuzzy approach, as opposed to traditional research methods. This study specifically analyzes shopping websites in Taiwan by consulting 12 experts and evaluating various sites through the Fuzzy TOPSIS method [20]. In this study, we use the TOPSIS methodology to evaluate the performance of four leading notebook computer original design manufacturers (ODMs) worldwide. The TOPSIS approach addresses the multi-attribute decision-making (MADM) problem by exploring alternatives in an n-dimensional geometric space defined by m points. An alternative is considered positive ideal if it is closest to the optimal solution (ie, minimizes the distance for each criterion) and negative ideal if it is far from the optimal solution (ie, maximizes the distance for each criterion). TOPSIS introduces an index called distance from the ideal solution, which helps identify the alternative that most closely approximates the positive ideal solution [21]. It defines ideal and negative ideal solutions when establishing a nonlinear model platform for distance measurement. This method is important for intuitive decision making involving weighted intervals. Initially, interval weights are calculated using function rules for interval values involving fuzzy attribute values. Next, the best and worst solutions are identified by a scoring function. The distance related to the fuzzy interval value is then defined, enabling the calculation of the distances between optimal and negative optimal solutions for each alternative. Using the TOPSIS method, relative sizes are calculated, which allows the ranking of projects based on their relative size [22]. This study presents a method for calculating the weight of the first general real number attribute within evidence theory. It establishes ideal and negative ideal solutions when building a nonlinear model platform for distance measurement. This method is important for intuitive decision making involving weighted intervals. Initially, interval weights are determined using function rules for interval values that include fuzzy attribute values. Then, the best and worst solutions are identified by a score function. The distance corresponding to the fuzzy interval value is then defined, enabling the calculation of the distances between optimal and negative optimal solutions for each alternative. By using the TOPSIS method, relative sizes are calculated, which allows the ranking of projects based on their relative size [23]. In this context, Gray theory helps in solving uncertain problems. To find the best solution, we use the technique of ordered prioritization by similarity to the best solution (TOPSIS). This paper compares TOPSIS with other methods, highlighting its importance for supplier ranking and selection. Although some methods share similarities with TOPSIS, they often overlook the evaluation of negatively graded alternatives. To address the concerns raised by Li et al. and Deng, Jadid et al. introduced a new technique based on TOPSIS principles [24

]. This paper presents an intuitive fuzzy multi-scale panel decision-making method to solve the supplier selection problem, specifically using TOPSIS. In supplier selection, decision makers often face difficulties in accurately defining the importance of criteria and the impact of alternatives due to the use of soft data. Intuitive fuzzy sets, introduced by Atanasov in 1986, provide a suitable solution to this challenge and are commonly used in decision making under uncertainty [25]. In team decision-making, effective integration of expert opinions during the evaluation process is critical. The Intuitive Fuzzy Weighted Average operator is used to adjust the opinions of all individual decision makers when evaluating the importance of criteria and alternatives. The TOPSIS method is a widely recognized approach to multi-attribute decision making that considers positive-ideal and negative-ideal solutions. Therefore, combining the TOPSIS method with intuitionistic fuzzy sets improves the potential for successful supplier selection [26]. One of the most commonly used multiple criteria decision making (MCDM) methods for employee selection is the fuzzy analytic hierarchy process. This approach involves establishing a set of criteria and assigning weights based on their importance. Applicants will be assessed against these criteria, resulting in a final score that reflects their significance. Another well-known method is the technique of ordered prioritization by similarity to the ideal solution (TOPSIS). This is consistent with a general human thought process where preferences are evaluated against ideals rather than personal criteria. To improve the performance of both methods, this paper proposes a combined fuzzy AHP-TOPSIS approach. A case study involving a company in Indonesia divided human resources functions into seven areas in order to select a manager. Seven managers were selected from a panel of five in each zone. The HR department established metrics as selection criteria, gathering input from five HR managers on the relative importance of each metric. By integrating their insights, we assessed how these managers' perspectives aligned with the organization's values, which were critical to the selection process [27]. The method addresses various levels of precision and uncertainty by evaluating the performance of alternatives against each criterion, with each decision maker's judgment informed by qualitative linguistic labels. This new system integrates input from decision makers, rather than relying on prior accumulation of linguistic data, and facilitates a comprehensive ranking of alternatives. The proposed system integrates the judgments of each decision maker. An example application in energy planning involves ranking energy policy alternatives [28]. Seven energy alternatives were evaluated based on nine criteria with input from three environmental and energy experts. Criteria weights were established using fuzzy AHP, and alternatives were ranked using TOPSIS. The proposed approach is compared with a modified Fuzzy Topsys method, demonstrating its advantages in managing the uncertainty and imprecision associated with linguistic evaluations [29]. In real-life scenarios, evaluating the suitability of plant locations often involves subjective criteria, with data and criteria weights usually expressed in linguistic terms. In addition, the available information is often ambiguous, requiring human judgment to navigate the inherent uncertainty in such decisions. Ratings and weights for all criteria are represented using fuzzy numbers [30].

3. RESULTS AND DISCUSSION

TABLE 1. Physics Informed Neural Networks

	Data Efficiency	Interpretability	Computational Costs	Incorporation of Physical Laws
Traditional Numerical Methods	20.10	33.25	91.45	42.23
Machine Learning Models	25.10	11.54	69.23	87.65
Hybrid Physics-ML Models	91.21	66.41	45.23	97.82
Gaussian Process Regression	65.23	98.24	91.41	65.24
Symbolic Regression	87.88	87.54	88.71	66.21

Physically informed neural networks (PINNs) combine the power of machine learning with the laws of physics and offer many advantages over traditional approaches. Using the TOPSIS method to evaluate different models, we evaluate the performance in four main criteria: data efficiency, interpretation, computational costs, and convergence of physical laws (Table 1). Traditional numerical methods scored moderately in interpretability (33.25), but were computationally expensive (91.45). Machine learning models are highly efficient in data utilization (25.10), lagging in interpretation (11.54). In contrast, hybrid physical-ML models excel, with high data efficiency (91.21), good interpretation (66.41) and strong adherence to physical laws (97.82). However, their computational costs (45.23) are significantly lower than traditional methods. Gaussian process regression models score well in interpretation (98.24) but suffer from high computational costs (91.41). Index regression achieves a balance between interpretability (87.54) and computational efficiency (88.71), with decent integration of physical laws (66.21). Overall, hybrid physical-ML models emerge as the most promising, effectively balancing these trade-offs.

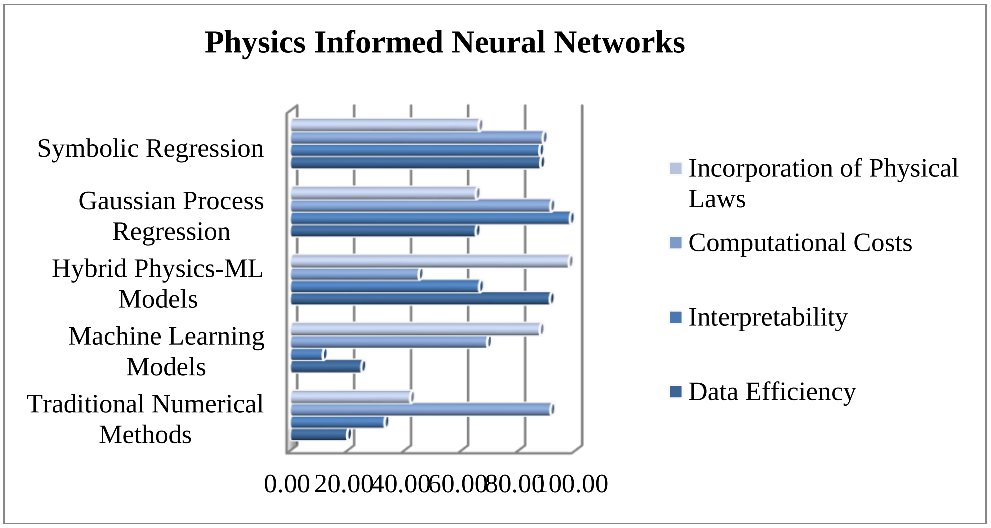

FIGURE 1. Physics Informed Neural Networks

Using the TOPSIS method, Figure 1 compares the performance of various models based on data efficiency, interpretability, computational costs, and incorporation of physical laws. Traditional numerical methods score high in computational costs (91.45) but low in data efficiency (20.10). Machine learning models show moderate data efficiency (25.10) and high incorporation of physical laws (87.65) but suffer in interpretability (11.54). Hybrid physics-ML models excel in data efficiency (91.21) and physical law integration (97.82), while maintaining reasonable computational costs (45.23). Symbolic regression and Gaussian Process Regression balance interpretability and computational efficiency but have varying data efficiency and costs.

TABLE 2. Normalized Data

	Data Efficiency	**Interpretability**	**Computational Costs**	**Incorporation of Physical Laws**
Traditional Numerical Methods	0.1376	0.2194	0.5159	0.2539
Machine Learning Models	0.1719	0.0762	0.3905	0.5269
Hybrid Physics-ML Models	0.6245	0.4382	0.2551	0.5880
Gaussian Process Regression	0.4466	0.6483	0.5156	0.3922
Symbolic Regression	0.6017	0.5777	0.5004	0.3980

Table 2 presents the normalized data using the TOPSIS method to compare models across four key criteria: data efficiency, interpretability, computational costs, and incorporation of physical laws. Traditional numerical methods show low data efficiency (0.1376) and moderate interpretability (0.2194), but they are computationally expensive (0.5159). Machine learning models perform better in data efficiency (0.1719) and excel in incorporating physical laws (0.5269), but their interpretability remains poor (0.0762). Hybrid physics-ML models lead in data efficiency (0.6245) and show strong performance in physical law incorporation (0.5880) while maintaining lower computational costs (0.2551) compared to other models. Gaussian Process Regression offers the highest interpretability (0.6483) but incurs significant computational costs (0.5156). Symbolic regression strikes a balance, with high data efficiency (0.6017) and interpretability (0.5777), but computational costs (0.5004) and physical law incorporation (0.3980) remain moderate. Overall, hybrid physics-ML models offer the most balanced trade-off, excelling in data efficiency and physical law adherence with lower computational burdens, making them the most promising approach for integrating physics-informed modeling and machine learning. Gaussian Process Regression and Symbolic Regression, while strong in interpretability, face challenges in balancing efficiency and costs.

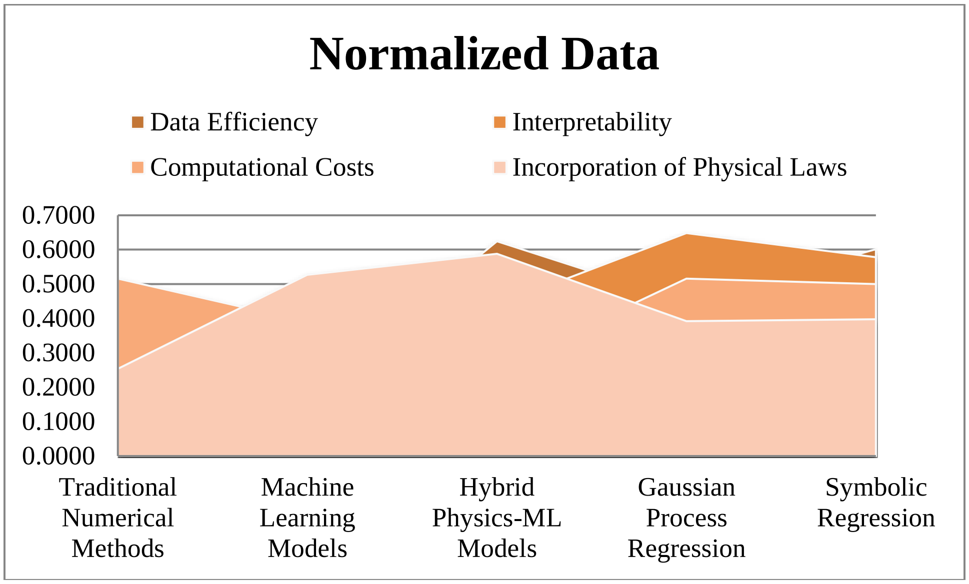

FIGURE 2. Normalized Data

Figure 2 presents normalized data using the TOPSIS method, comparing models on key criteria. Traditional numerical methods exhibit low data efficiency (0.1376) and moderate interpretation (0.2194), but high computational costs (0.5159). Machine learning models are highly efficient in data utilization (0.1719) and excel in combining physical laws (0.5269), although interpretability is poor (0.0762). The hybrid physical-ML models show strong performance in data efficiency (0.6245) and physical law convergence (0.5880), while maintaining low computational costs (0.2551). Exponential and Gaussian process regression models provide balanced interpretation but face moderate computational challenges.

TABLE 3. Weight ages

Weight			
0.25	0.25	0.25	0.25
0.25	0.25	0.25	0.25
0.25	0.25	0.25	0.25
0.25	0.25	0.25	0.25
0.25	0.25	0.25	0.25

Table 3 shows the weight ages used in the TOPSIS method, assigning equal importance (0.25) to each of the four evaluation criteria: data efficiency, interpretation, computational costs, and integration of physical laws. By applying these equal weightings, the analysis considers each criterion equally important to the overall performance of the models being evaluated. This balanced weighting approach ensures that no single criterion dominates the decision-making process, enabling a fair and comprehensive comparison. In practical terms, this means that models that excel at one criterion, such as computational efficiency, may not perform better than others that are stronger at interpretation or following a physical law. As a result, models such as hybrid physical-ML models that show strong balance in multiple dimensions are likely to emerge as optimal choices in this context. Meanwhile, approaches such as computationally expensive traditional numerical methods or no descriptive machine learning models may be less favorable due to this asymmetric estimation scheme. Ultimately, the equal weighting method facilitates a thorough comparison encouraging the selection of models that strike the best balance between performance, interpretation, computational demands, and alignment with physical laws, as seen in models such as mixed physical-ML or index regression approaches.

TABLE 4. Weighted normalized decision matrix

	Weighted normalized decision matrix			
Traditional Numerical Methods	0.0344	0.0549	0.1290	0.0635
Machine Learning Models	0.0430	0.0190	0.0976	0.1317
Hybrid Physics-ML Models	0.1561	0.1096	0.0638	0.1470
Gaussian Process Regression	0.1117	0.1621	0.1289	0.0980
Symbolic Regression	0.1504	0.1444	0.1251	0.0995

Table 4 presents the weighted normalized result matrix using the TOPSIS method, with the normalized values from previous analyzes adjusted by equal weights (0.25) assigned to each criterion. This matrix factors the strengths of different models in terms of their relative performance, data capacity, interpretation, computational costs, and convergence of physical laws. Traditional numerical methods show low performance in all criteria, especially data efficiency (0.0344) and interpretation (0.0549), while their computational costs are high (0.1290). Machine learning models score moderately in computational efficiency (0.0976) and excel in combining physical laws (0.1317), but their interpretability is significantly lower (0.0190). Hybrid physical-ML models outperform other approaches with strong scores in data efficiency (0.1561) and physical law convergence (0.1470), low computational costs (0.0638), making them a well-rounded choice. Gaussian process regression exhibits high explanatory power (0.1621), but computational costs remain high (0.1289). The index shows regression balance, excelling in data fit (0.1504) and interpretability (0.1444), although it is moderate in other criteria. Overall, the hybrid physical-ML models and symbolic regression are more balanced, providing strong performance in all categories with lower computational costs compared to traditional approaches.

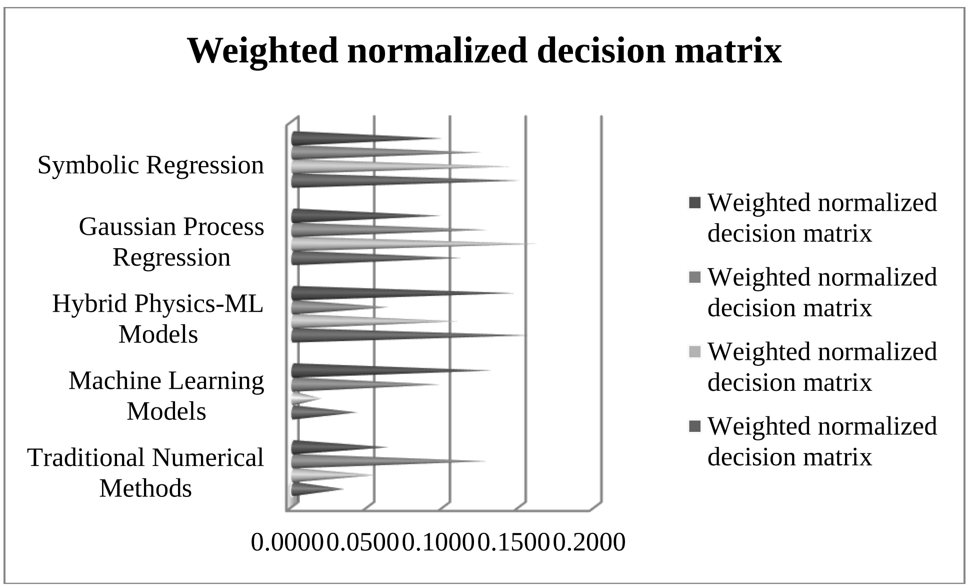

FIGURE 3. Weighted normalized decision matrix

Figure 3 presents the weighted normalized result matrix using the TOPSIS method, highlighting the model performance after applying equal weights to criteria. Traditional numerical methods perform poorly in all categories, especially data efficiency (0.0344) and interpretation (0.0549). Machine learning models excel at combining physical laws (0.1317) but have low interpretability (0.0190). The hybrid physical-ML models show strong overall performance, excelling in data fit (0.1561) and physical law adherence (0.1470). Gaussian process regression and exponential regression perform best in description, but hybrid physical-ML models provide a better balance across all criteria.

TABLE 5. Positive and Negative Matrix

Positive Matrix				Negative matrix			
0.1561	0.1621	0.0638	0.0635	0.0344	0.0190	0.1290	0.1470
0.1561	0.1621	0.0638	0.0635	0.0344	0.0190	0.1290	0.1470
0.1561	0.1621	0.0638	0.0635	0.0344	0.0190	0.1290	0.1470
0.1561	0.1621	0.0638	0.0635	0.0344	0.0190	0.1290	0.1470
0.1561	0.1621	0.0638	0.0635	0.0344	0.0190	0.1290	0.1470

Table 5 presents the positive and negative matrices obtained from the TOPSIS method, where the positive matrix contains the best performing values for each criterion and the negative matrix captures the worst performing values. The positive matrix highlights the best performance in terms of data efficiency, interpretation, computational costs, and convergence of physical laws, with values such as 0.1561 and 0.1621 reflecting strong data efficiency and interpretation, respectively. These values represent models such as hybrid physical-ML models and Gaussian process regression that excel in these areas. On the other hand, the negative matrix shows poor performance for each criterion, with low values such as 0.0344 for data efficiency and 0.0190 for interpretation, while traditional numerical methods and machine learning models struggle in these dimensions. The negative matrix emphasizes trade-offs in computational costs (0.1290) and physical law convergence (0.1470), where less efficient models incur higher costs. By comparing the two metrics, models such as hybrid physical-ML and index regression stand out for effectively balancing these criteria. The positive matrix represents the target, while the negative matrix serves as a criterion for the lowest acceptable performance, guiding the selection of optimal models in various applications.

TABLE 6. Final Result of Physics Informed Neural Networks

	SI Plus	Si Negative	Ci	Rank
Traditional Numerical Methods	0.1748	0.0909	0.3421	4
Machine Learning Models	0.1977	0.0359	0.1537	5
Hybrid Physics-ML Models	0.0987	0.1651	0.6259	3
Gaussian Process Regression	0.0861	0.1698	0.6635	2
Symbolic Regression	0.0735	0.1773	0.7070	1

Table 6 presents the final results of Physically Informed Neural Networks (PINNs) evaluated using the TOPSIS method, each model's Si Plus (positive best solution), Si Negative (negative best solution), Ci (closeness relative to the best solution) and ranking. Traditional numerical methods achieve a Si plus of 0.1748 and a Si negative of 0.0909, resulting in a moderate Ci value of 0.3421, ranking 4th. Machine learning models ranked lowest (5th) with a Ci of 0.1537, reflecting their weak interpretation and high computational costs. The hybrid physical-ML models perform remarkably well, with a negative Si of 0.1651 and a Ci of 0.6259, ranking 3rd due to their strong data capability and physical frame integration. The Gaussian process regression ranks 2nd with a Ci of 0.6635, which exhibits the best description and demonstrates a good balance between computational costs and adherence to physical law. Index regression tops the rankings with the highest Ci value of 0.7070, making it the best overall performance due to the balance between data efficiency, interpretation and computational costs. Overall, exponential regression and Gaussian process regression stand out as the top two models, with exponential regression outperforming all key criteria. Hybrid physical-ML models also offer robust performance, particularly in data efficiency and physical law integration.

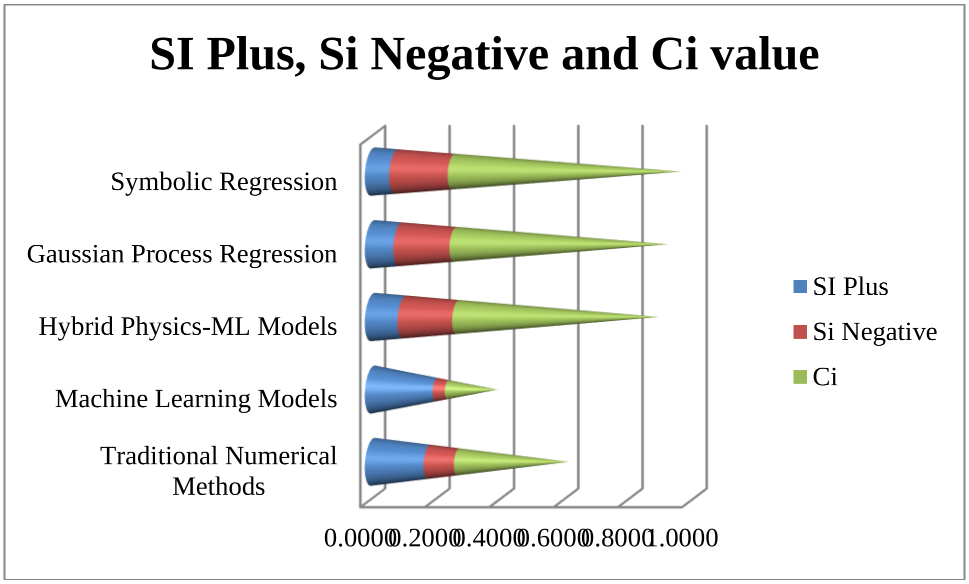

FIGURE 4. Result of Physics Informed Neural Networks

Figure 4 presents the evaluation results of Physically Informed Neural Networks (PINNs) using the TOPSIS method. Ci values indicate the closeness of the models to the best solution, where higher values indicate better performance. Index regression ranks first with 0.7070 Ci, excelling in data efficiency, interpretation, and computational costs. The Gaussian process regression continues with a Ci of 0.6635, showing a robust interpretation. Hybrid physical-ML models ranked third with 0.6259 Ci, balancing efficiency and physical law convergence. Traditional numerical methods and machine learning models are ranked low due to high computational costs and low interpretability.

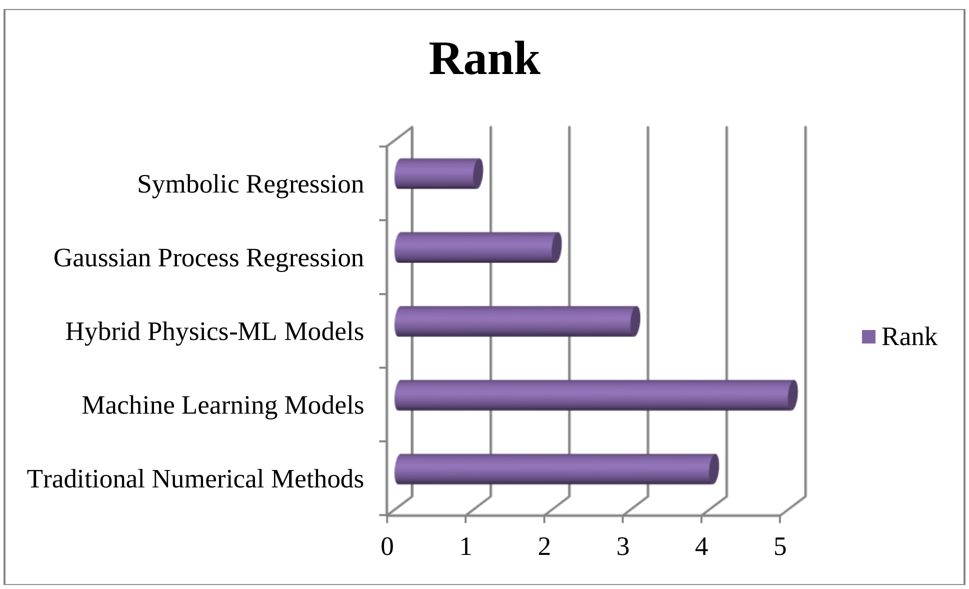

FIGURE 5. Rank

Figure 5 illustrates the ranking of models based on TOPSIS method evaluation. Index Regression tops the list, earning the 1st rank due to its balanced performance across all criteria. Gaussian process regression follows closely in 2nd place, exhibiting strong explanatory power and efficiency. Hybrid physical-ML models ranked 3rd, with good data efficiency and following physical laws. Traditional numerical methods come in at number 4, primarily because of high computational costs. Machine learning models ranked the lowest at number 5, struggling in both interpretability and performance. This ranking highlights the varying strengths and weaknesses of each approach in the context of physics-informed neural networks.

4. CONCLUSION

Based on the detailed analysis presented in this study, we can draw several important conclusions about physics-informed neural networks (PINNs) and related methods in scientific computing and engineering problems. Evaluation of different models using the TOPSIS method revealed significant insights into their relative strengths and weaknesses in key criteria: data capacity, interpretation, computational costs, and convergence of physical laws. Indexed regression emerged as the best ranking approach exhibiting well-balanced performance across all criteria. Its high data capacity and interpretability, combined with reasonable computational costs and convergence of physical laws, make it a versatile choice for a wide range of applications. Gaussian process regression ranked second, particularly excelling in interpretation. This suggests its potential for applications where understanding the model's decision-making process is critical, such as scientific research or high-stakes engineering decisions. Hybrid physical-ML models showed strong performance, especially in data efficiency and convergence of physical laws. This approach effectively combines the strengths of traditional physics-based modeling with the flexibility and learning capabilities of machine learning, making it well suited for complex physical systems. Traditional numerical methods, although valuable in many contexts, showed limitations in data efficiency and interpretation compared to newer approaches. However, the strong convergence of physical laws highlights their continuity in some domains. Machine learning models, despite their widespread use and success in many fields, rank very low in this comparison. This suggests that for problems where physical laws play an important role, pure data-driven approaches may fall short without integration of field knowledge. This study underscores the importance of balancing multiple criteria when selecting modeling approaches for physics-informed problems. Although no single method dominates all criteria, the primary approaches (code regression, Gaussian process regression, and hybrid physical-ML models) offer different strengths that can be leveraged depending on the specific needs of the problem. The results highlight the trend towards hybrid approaches that combine the best aspects of traditional physics-based modeling with modern machine learning techniques. These methods promise to address the limitations of pure data-driven or pure physics-based approaches, providing improved data efficiency, interpretation, and adherence to the laws of physics while managing computational costs. In conclusion, this study provides valuable insights for researchers and practitioners in scientific computing, engineering and related fields. It suggests that the future of modeling complex physical systems lies in approaches that can effectively integrate field knowledge with data-driven learning. As these methods continue to evolve, we can expect further improvements in our ability to model, understand, and predict complex physical phenomena in a wide range of applications.

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