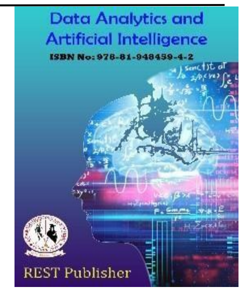
Data Analytics and Artificial Intelligence

Vol: 5(1), 2025

REST Publisher; ISBN: 978-81-948459-4-2

Website: http://restpublisher.com/book-series/daai/

DOI: https://doi.org/10.46632/daai/5/1/11



Novel Score Function For Ranking of Intuitionistic Fuzzy Set

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Abstract: *The score function is a crucial instrument for ranking of intuitionistic fuzzy numbers (IFNs). It assists decision-makers in more precise data processing and decision-making in uncertain conditions. This study presents a novel score function of IFNs. By using proposed score function, we develop a new method to compare the IFNs. Additionally, we assess the validity of the proposed score function by examining additional desirable properties. The proposed score function can rectify the deficiencies of the existing score functions of IFNs, which are unable to compare the IFNs. Ultimately, data trials are conducted that confirm the efficacy of the proposed score function, resulting in the conclusion that it surpasses the existing score functions of IFNs.*

Keywords: *Score function; IFNs; Ranking; Fuzzy set.*

1. INTRODUCTION

Fuzzy set (FS) [18] theory, established by Prof. Zadeh in 1965, represents a pivotal advancement in the mathematical representation of uncertainty and imprecision. In contrast to conventional set theory, which relies on a binary membership paradigm where elements are either included in a set or excluded, fuzzy set theory allows for partial membership. This method facilitates a more accurate representation of genuine situations, frequently characterized as ambiguous and uncertain. In decision-making processes, fuzzy set theory is advantageous for representing judgments and preferences that are frequently subjective. Consequently, it encapsulates the ambiguity and indistinctness inherent in human cognitive processes. Decision makers enhance the quality and reliability of their decisions with FS-based methodologies. These methodologies are extensively employed in fields such as economics, finance, and resource management, where decision-makers encounter uncertainty, as well as ambiguous and conflicting information.

Although FS theory presents significant advantages, it also has limits, particularly in settings characterized by elevated levels of uncertainty. To address these constraints, Atanassov [1] introduced intuitionistic fuzzy sets (IFSs) in 1986, incorporating a degree of non-membership (NMD) in addition to the degree of membership (MD). In IFS, each element is defined by a MD, a NMD, and a hesitancy degree (HD) that indicates the ambiguity in determining the element's membership. This supplementary dimension offers a broader depiction of uncertainty, hence facilitating improved management of intricate decision-making circumstances. Consequently, numerous researchers have explored IFSs and applied them to solve various decision-making problems [3, 4, 5, 6, 8, 9, 10, 11, 13, 17, 19]. Chen and Tan [3] introduced a score function for IFNs and later it was extended by Wu and Chiclana [17]. Kumar and Garg [13] proposed an extended TOPSIS method based on set pair analysis and its connection number for ranking alternatives under IF environments. Kumar and Chen [12] developed an enhanced IF weighted arithmetic aggregation operator (AO) based on Einstein norms. Ejegwa and Agbetayo [6] proposed a novel similarity measure and distance measure for IFSs and applied them to solve various decision-making problems. Tripathi et al [16] presented a new divergence measure and a generalized score function for IFSs. Kumar et al [14] proposed an entropy measure for linguistic IFSs. Hussain et al [9] introduced prioritized AOs for IFSs based on Sugeno-Weber norms and applied them to evaluate the suitable digital security technique. Senapati et al [15] proposed power aggregation operators AOs for IFSs based on Aczel-Alsina norms and applied them to select the most suitable sustainable transportation sharing practice. Bhardwaj et al [2] proposed an entropy measure for IFSs to determine attribute weights and address MADM problems. Zhang et al [19] proposed a extended TODIM method employing the proposed score function under IF environment. Garg and Kumar [7] introduced a possibility degree measure for IFSs with applications in MADM.

In this paper, we observe that the existing score functions given in [3, 17], and [19], have limitation that they are unable to rank the IFNs in certain scenarios. Therefore, in order to overcome these limitations, there is a need to develop a new score function. Therefore, this study proposes a novel score function for IFNs to address the deficiencies of existing score functions in the effective ordering of IFNs. The new score function enhances accurate ranking and decision-making under ambiguity while satisfying essential qualities required for dependable evaluation. Furthermore, the results of the comparative analysis confirm the superiority of the proposed score function, demonstrating its ability to surpass the constraints of existing scoring functions.

The remaining paper is structured in the subsequent manner: In Section 2, we provide a brief outline of the essential knowledge of IFSs and mentions some existing score functions. In Section 3, we introduce a new score function for the ranking of IFNs and prove several salient properties. Section 4 presents a comparative analysis with the existing score functions, demonstrating the effectiveness of the proposed score function. Finally, Section 5 presents the conclusion.

2. PRELIMINARIES

Definition 1. [1] The IFS M in the universe of discourse X is defined as:

$$M = \{\langle x, \alpha_M(x), \beta_M(x)\rangle | x \in X\}, \tag{1}$$

where $\alpha_M(x): X \to [0,1]$ indicates the MD and $\beta_M(x): X \to [0,1]$ indicates the NMD of $x \in X$, where $0 \leq \alpha_M(x) + \beta_M(x) \leq 1$. The HD of an element $x \in X$ is $\pi_M(x) = 1 - \alpha_M(x) - \beta_M(x)$. An IFN is expressed as $\langle \alpha_M, \beta_M \rangle$.

Definition 2. Let $M = \langle \alpha_M, \beta_M \rangle$ be an IFN, then some of the existing score functions are stated as:

(i) Chen and Tan [3] score function:

$$S_1(M) = \alpha_M - \beta_M, \tag{2}$$

where, $S_1(M) \in [-1,1]$.

(ii) Wu and Chiclana [17] score function:

$$S_2(M) = \alpha_M + \frac{\pi_M}{2}, \tag{3}$$

where, $S_2(M) \in [0,1]$.

(iii) Zhang et al [19] score function:

$$S_3(M) = \frac{\alpha_M}{\alpha_M + \beta_M}, \tag{4}$$

where, $S_3(M) \in [0,1]$.

Definition 3. [3] Let $M_1 = \langle \alpha_1, \beta_1 \rangle$ and $M_2 = \langle \alpha_2, \beta_2 \rangle$ be any two IFNs, then we have

(i) If $score(M_1) > score(M_2) \Longrightarrow M_1 \succ M_2$.
(ii) If $score(M_1) = score(M_2) \Longrightarrow M_1 \sim M_2$.

3. NEW SCORE FUNCTION FOR INTUITIONISTIC FUZZY NUMBERS (IFNS)

In this section, we present a novel score function for ranking of IFNs and provide the desirable properties of the proposed score function.

Definition 4. Let $M = \langle \alpha_M, \beta_M \rangle$ be an IFN, then the score function $S(M)$ is stated as:

$$S(M) = \frac{1}{3}(2\alpha_M - \beta_M(1 + \pi_M) + 1), \quad (5)$$

where α_M, $\beta_M \in [0,1]$, and $\pi_m = 1 - \alpha_M - \beta_M$.

Example 1. Let $M = \langle 0.2,0.3 \rangle$ be an IFN, then by using Eq. (5), the score function of M is obtained as follows:

$$S(M) = \frac{1}{3}(2(0.2) - 0.3(1 + \pi_M) + 1)$$
$$= \frac{1}{3}(2(0.2) - 0.3(1 + 0.5) + 1)$$
$$= 0.316.$$

Property 1. Let $M = \langle \alpha_M, \beta_M \rangle$ be a IFN, then the score function $S(M) \in [0,1]$.

Proof. For any IFS, we have $\alpha_M \geq 0$, and $\beta_M \leq 1$, which implies that $\pi_M \geq 0$, $2\alpha_M \geq 0$, and $-\beta_M(1 + \pi_M) + 1 \geq 0$. Hence,

$$2\alpha_M - \beta_M(1 + \pi_M) + 1 \geq 0$$
$$\Longrightarrow \frac{2\alpha_M - \beta_M(1+\pi_M)+1}{3} \geq 0$$
$$\Longrightarrow S(M) \geq 0.$$

Similarly, for any IFS, we have $\alpha_M \leq 1$, and $\beta_M \geq 0$, which implies that $\pi_M \geq 0$, $2\alpha_M \leq 2$, and $-\beta_M(1 + \pi_M) \leq 0$. Hence,

$$2\alpha_M - \beta_M(1 + \pi_M) \leq 2$$
$$\Longrightarrow 2\alpha_M - \beta_M(1 + \pi_M) + 1 \leq 3$$
$$\Longrightarrow \frac{2\alpha_M - \beta_M(1+\pi_M)+1}{3} \leq 1$$
$$\Longrightarrow S(M) \leq 1.$$

Thus, $0 \leq S(M) \leq 1$.

Property 2. If $M = \langle 1,0 \rangle$, then $S(M) = 1$ and if $M = \langle 0,1 \rangle$, then $S(M) = 0$.

Proof. When $M = \langle 1,0 \rangle$, then by using Eq. (5), we obtain

$$S(M) = \frac{2.1 - 0.(1+\pi_M)+1}{3}$$
$$= \frac{2-0+1}{3}$$
$$= 1.$$

Similarly, when $M = \langle 0,1 \rangle$, then by using Eq. (5), we obtain

$$S(M) = \frac{2.0 - 1.(1+\pi_M)+1}{3}$$
$$= \frac{0 - 1.(1+(1-\alpha_M-\beta_M))+1}{3}$$
$$= \frac{0 - 1.(1+(1-0-1))+1}{3}$$
$$= 0.$$

Property 3. Let $M = \langle \alpha_M, \beta_M \rangle$ be an IFN, then the score function $S(M)$ monotonically increases with α_M and monotonically decreases with β_M.

Proof. Differentiate partially Eq. (5) with respect to α_M and β_M, respectively, we obtain,

$$\frac{\partial S}{\partial \alpha_M} = \frac{1}{3}(2 + \beta_M),$$
$$\frac{\partial S}{\partial \beta_M} = \frac{-1}{3}(2 - \alpha_M - 2.\beta_M).$$

Since, α_M, $\beta_M \in [0,1]$, so we have, $\frac{\partial S}{\partial \alpha_M} \geq 0$ and $\frac{\partial S}{\partial \beta_M} \leq 0$. Hence, the score function $S(M)$ monotonically increases as α_M increases and monotonically decreases as β_M increases.

Theorem 1. According to Definition 4, for any two IFNs $M_1 = \langle \alpha_1, \beta_1 \rangle$ and $M_2 = \langle \alpha_2, \beta_2 \rangle$, we have if $S(M_1) > S(M_2) \Rightarrow M_1 \succ M_2$.

4. ADVANTAGES OF THE PROPOSED SCORE FUNCTION OVER THE EXISTING SCORE FUNCTIONS

In this section, we presents the advantages of the proposed score function of IFN and shortcomings of the existing score functions defined in [3, 17], and [19].

Example 2. Let $M_1 = \langle 0,0.4 \rangle$ and $M_2 = \langle 0,0.5 \rangle$ be two IFNs. We compute the score functions S_1, S_2, S_3 and S by using Eqs. (2), (3), (4), and (5), respectively, to compare these IFNs, and the results are given in Table 1.

TABLE 1. A comparison of IFNs using score functions S_1, S_2, S_3 and S for Example 2

Score functions	Score values	Ranking
S_1 [3]	$S(M_1) = -0.4, S(M_2) = -0.5$	$M_1 \succ M_2$
S_2 [17]	$S(M_1) = 0.3, S(M_2) = 0.25$	$M_1 \succ M_2$
S_3 [19]	$S(M_1) = S(M_2) = 0$	$M_1 \sim M_2$
Proposed score function (S)	$S(M_1) = 0.12,\ S(M_2) = 0.083$	$M_1 \succ M_2$

It is evident from Table 1, that the score function S_3 introduced by Zhang et al [19] get same value for IFNs M_1 and M_2, that is, $S(M_1) = 0$ and $S(M_2) = 0$ and hence, fails to distinguish between them. However, the score functions S_1 [3], S_2 [17] , and the proposed score function S, obtain different score values, indicating that $M_1 \succ M_2$.

Example 3. Let $M_1 = \langle 0.12,0.28 \rangle$ and $M_2 = \langle 0.3,0.46 \rangle$ be two IFNs. We compute the score functions S_1, S_2, S_3 and S by using Eqs. (2), (3), (4), and (5), respectively, to compare these IFNs, and the results are given in Table 2.

TABLE 2. A comparison of IFNs using score functions S_1, S_2, S_3 and S for Example 3

Score functions	Score values	Ranking
S_1 [3]	$S(M_1) = S(M_2) = -0.16$	$M_1 \sim M_2$
S_2 [17]	$S(M_1) = S(M_2) = 0.42$	$M_1 \sim M_2$
S_3 [19]	$S(M_1) = 0.3, S(M_2) = 0.39$	$M_1 \prec M_2$
Proposed score function (S)	$S(M_1) = 0.264,\ S(M_2) = 0.34$	$M_1 \prec M_2$

It is evident from Table 2 that the score functions S_1 [3] and S_2 [17] obtain same values for IFNs M_1 and M_2, and therefore are unable to distinguish between them. However, the score function S_3 [19] and the proposed score function S yield different score values, indicating that $M_1 \prec M_2$.

5. CONCLUSION

In this work, a new score function for intuitionistic fuzzy numbers (IFNs) has been proposed to compensate for the shortcomings of current methods in effectively ordering of IFNs. The new score function not only facilitates more reliable ranking and decision-making with uncertainty but also fulfills important desirable properties necessary for reliable evaluation. A new method for ranking of IFNs based on the new score function has been established and implemented, proving its efficacy and applicability. Comparative studies and trials of data verify the excellence of the presented strategy further, ascertaining it against outperforming the limitations of current score functions. This development benefits fuzzy decision-making immensely by improving the accuracy and reliability of IFN comparison.

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