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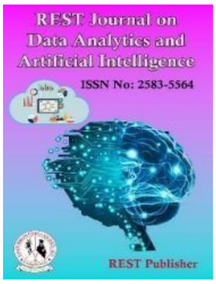

$\alpha G\mu$-Homemorphism In Topological Spaces

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Abstract: *The perception of an -Generalized -closed sets in topological spaces and also derived their homeomorphism. Relationships are established between αGμ- homeomorphism in topological spaces containing the class of homeomorphisms.*
Key words: *αGμ -continuous, αGμ -irresolute and αGμ- homeomorphism.*

1. INTRODUCTION

Njastad[1] was presented and examined the concept of α-open sets. By using α -open sets Mashhour et al.[13] defined and deliberate the concept of α-closed sets, α-closure of a set, α- continuity and α-closedness in topology. Levine [2] introduced g-closed sets and studied their most important belongings. M.K.R.S.Veera kumar[24] introduces generalized μ-closed sets in topological spaces. R. Devi.et.al [6]. established idea Semi-generalized homeomorphism and generalized semi-homeomorphism. A innovative class of set name as α *Generalized* μ-closed sets introduced by R.Devi and V.Vijayalakshmi[25] and study some application of αGμ -closed sets. In this article, we familiarized and examined the notion of αGμ -homeomorphism.

2. Preliminaries

Throughout this article the characterize topological spaces (H, ζ), (J, γ) and (R, λ) on which no leaving proverbs are supposed unless otherwise acknowledged. For subset B and a space (H, ζ), cl(B), int(B) and C(B) mean the closure of B, interior of B and supplement of B in H separately. P(H) represents the power set of H.
Here we remembrance the subsequent definitions, which will be recycled frequently throughout this article.

Definition 2.1

A map define by n: (H, ζ) → (J, γ) is called αGμ-continuous function if each closed set K of (J, γ), then $n^{-1}(K)$ is αGμ - closed in (H, ζ).

Definition 2.2

A map defines by n: (H, ζ) → (J, γ) is called αGμ -irresolute function if each closed set K of (J, γ), then $n^{-1}(K)$ is αGμ - closed in (H, ζ).

3. $\alpha G\mu$ -homeomorphism and their group structure.

We introduce the following definition.

Definition 3.1

A mapping function n is called a $\alpha G\mu_C$- homeomorphism (resp. $\alpha g\mu -$ homeomorphism), if n is bijective and n and n^{-1} are $\alpha G\mu$ - irresolute (resp. $\alpha G\mu$-continuous).

Theorem 3.2

Each $\alpha G\mu_C$- homeomorphism is $\alpha G\mu$- homeomorphism.

Proof

First consider map n: (H, ζ) → (J, γ), K be closed set and $\alpha G\mu$ - closed in (J, γ). Thus n^{-1} is $\alpha G\mu_C$ -closed in (H, ζ). Thus, n is $\alpha G\mu_C$ - irresolute. Then $n^{-1}(K)$ is $\alpha G\mu_C$ - closed in (H, ζ). Hence n is $\alpha G\mu$- continuous map. The proof of the $\alpha G\mu$ - continuous map for n^{-1} is similar to the above.

The converse of the above theorem requirement false through the resulting example.

Example 3.3

Consider H=J= {s, t, u} through ζ= {H, φ, {s}, {s, u}} and γ = {J, φ, {s, t}}.

Let n: (H, ζ) → (J, γ) be defined as n(s)=s, n(t)=u and n(u)=t.

Here n and n^{-1} are not a $\alpha G\mu$-irresolute map.

Hence n is $\alpha G\mu_C$–homeomorphism but not $\alpha G\mu$- homeomorphism,

Theorem 3.4

Each homeomorphism is $\alpha G\mu$- homeomorphism.

Proof

Define the mapping n: (H, ζ) → (J, γ) be a homeomorphism. By definition each continuous map is $\alpha G\mu$-continuous map and each closed set is $\alpha G\mu$ -closed, we accomplish that n is $\alpha G\mu$- homeomorphism.

The following example requirement false through the converse of the above theorem.

Example 3.5

Define H=J= {s, t, u} through ζ= {H, φ, {s}, {s, u}} and γ = {J, φ, {t, u}}.

Let n: (H, ζ) → (J, γ) be defined as n(s)=u, n(t)=s and n(u)=t.

Now n is not continuous map. Then n is $\alpha G\mu$–homeomorphism but not homeomorphism.

Theorem 3.6

Each $\alpha G\mu$ - homeomorphism is g-homeomorphism.

Proof

Let us define mapping n: (H, ζ) → (J, γ) be a αGμ-homeomorphism. We know that each αGμ -continuous map is a $g-continuous$ map and each αGμ -closed set is $g-closed$, we achieve that n is a $g-$ homeomorphism.

The resulting example requirement false through the converse of the above theorem.

Example 3.7

Consider H=J= {s, t, u} through ζ= {H, φ, {t}, {t, u}} and γ = {J, φ, {t, u}}.

Let n: (H, ζ) → (J, γ) be defined as n(s)=t, n(t)=u and n(u)=s.

The result n and n $^{-1}$ are not a αGμ-continuous map.

Since n is a g–homeomorphism but not an αGμ- homeomorphism.

Theorem 3.8

Each αGμ- homeomorphism is a gs-homeomorphism.

Proof

Consider the mapping n: (H, ζ) → (J, γ) be a αGμ-homeomorphism. By definition, each αGμ-continuous map is a $gs-continuous$ and each αGμ -closed set is a $gs-closed$, hence prove that n is a gs-homeomorphism.

The resulting example requirement false through the converse of the above theorem.

Example 3.9

Consider H=J= {s, t, u} through ζ= {H, φ, {s}, {s, u}} and γ = {J, φ, {s}}.

Let n: (H, ζ) → (J, γ) be defined as n(s)=s, n(t)=u and n(u)=t.

Now n is not αGμ-continuous map. Because n is gs-homeomorphism but not an αGμ- homeomorphism,

Theorem 3.10

Each αGμ- homeomorphism is gsp-homeomorphism.

Proof

Let us consider n be a αGμ-homeomorphism by n: (H, ζ) → (J, γ). By definition each αGμ -continuous map is $gsp-continuous$ and each αGμ -closed set is $gsp-closed$, then desired that n is gsp-homeomorphism.

The resulting example requirement false through the converse of the above theorem.

Example 3.11

Consider H=J= {s, t, u} through ζ= {H, φ, {s}, {s, u}} and γ = {J, φ, {s}}.

Let n: (H, ζ) → (J, γ) be defined as n(s)=s, n(t)=u and n(u)=t.

Here n is not αGμ-continuous map, thus n is gsp -homeomorphism but not an αGμ- homeomorphism.

Theorem 3.12

Each αGμ- homeomorphism is $\alpha g -$-homeomorphism.

Proof

Let n: (H, ζ) → (J, γ) be a αGμ -homeomorphism. We know that, each αGμ -continuous map is $\alpha g - continuous$ and each αGμ - closed set is $\alpha g - closed$, thus n is $\alpha g -$homeomorphism.

The resulting example requirement false through the converse of the above theorem.

Example 3.13

Consider H=J= {s, t, u} through ζ= {H, φ, {s}, {s, u}} and γ = {J, φ, {t, u}}.

Let n: (H, ζ) → (J, γ) be defined as n(s)=u, n(t)=t and n(u)=s.

The result n is not αGμ-continuous map and not an αGμ- homeomorphism.

Hence n is $\alpha g -$ homeomorphism.

Theorem 3.14

Each αGμ- homeomorphism is pre-semi-homeomorphism.

Proof

A mapping define n: (H, ζ) → (J, γ) be a αGμ -homeomorphism. Since, each αGμ -continuous map is $pre - semi - continuous$ and each αGμ -closed set is pre-semi-closed. We accomplish n is pre-semi-homeomorphism.

The resulting example requirement false through the converse of the above theorem.

Example 3.15

Consider H=J= {s, t, u} through ζ= {H, φ, {s}, {s, u}} and γ = {J, φ, {s, t}}.

Let n: (H, ζ) → (J, γ) be defined as n(s)=s, n(t)=t and n(u)=u.

Then n is pre-semi-homeomorphism but not an αGμ- homeomorphism. Hence n is not αGμ-continuous map.

Theorem 3.16

Every $g^{\#} -$ homeomorphism is αGμ -homeomorphism.

Proof

Let n: (H, ζ) → (J, γ) be a $g^{\#} -$homeomorphism. By definition each $g^{\#} -$continuous map is αGμ-continuous and each $g^{\#} -$closed map is αGμ -closed, we settle that n is αGμ -homeomorphism.

The resulting example requirement false through the converse of the above theorem.

Example 3.17

Consider H=J= {s, t, u} through ζ= {H, φ, {s}, {t, u}} and γ = {J, φ, {s, t}}

Let n: (H, ζ) → (J, γ) be defined as n(s)=s, n(t)=t and n(u)=u.

Here the map n is αGμ- homeomorphism. But n is not $g^{\#}$ – homeomorphism, since the map n is not αGμ-continuous map.

2. CONCLUSION

In this article, perception of an -Generalized -closed sets in topological spaces and their homeomorphism are derived. Further, we discussed αGμ- homeomorphism in topological spaces containing the class of homeomorphisms.

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