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Accurate Characterization of Crack in Beam-Like Structures Using Square of Curvature Mode Shape: Without Baseline Data

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Abstract: *In this paper, a new modal parameters-based damage detection method has been proposed. This method is based on accurately composition of undamaged mode profiles from the damaged mode profiles of beam structure. For that, curve fitting (CF) analysis was used to extract the undamaged mode profiles from damaged ones. Curvature mode shape method is presented to demonstrate proposed approach without use of base line data as a reference. To illustration of this method, numerical and experimental modal analysis is performed on two aluminum cantilever beams containing single and double cracks to examine the location and quantification of different damage scenarios. First five modal frequencies and mode profiles are employed to assess the effectiveness of the proposed method. Moreover, to validate the accuracy of the proposed approach in damage identification, established results have been balanced with another experimental based results. From the results, it can be seen that damage scenarios of the two beams are quite different the chosen method is precisely identifying the location and quantification of the damage in beam structures. It is concluded that composed mode profiles by the proposed approach can be conveniently used as reference data for the damage identification purpose. Further, an influence of mode order on damage identification also investigated in the present analysis.*

Keywords: *Beam-like structure, mode profiles, Crack detection, Mode shape curvature, Experimental Modal Analysis, discrete Wavelet Transform.*

INTRODUCTION

The structural damage detection in civil, mechanical, marine, and aerospace structures is plays essential role not only in the health analysis and estimation of its life span but also estimating the eventual expenditure and planning for overhaul appropriately. Structural Health Monitoring (SHM) is a non-destructive damage detection strategy of structures for analyzing its condition based on the structure's response. Recent decades SHM plays essential role in assuring the structural integrity. In this scenario, damage is defined as the changes in dynamic parameters of vibrational structure or boundary conditions of structure which suspiciously influence the working of structure. Damage is also defined as changes in the geometries which lead to inadmissible internal stresses, excessive deflections or vibrations in the structural components/system. These instabilities are due to the internally generated cracks, corrosive spots, loosening of mated parts in a system, ageing of structure and fatigue, etc. In past few years, many researchers have been developed sophisticated dynamic damage identification methods, which the related literature is substantial. The basic idea of elemental or essential assertion in the vibration based damage detection (VBDD) approaches is that the changes in the physical properties such as mass, damping, stiffness, etc. will causes the alteration of the structure's dynamic responsive modal parameters such as modal frequencies, mode profiles or modal displacements, mode shape derivatives, modal strain energy changes, and perturbations in the propagating waves, etc. have been found in the literature [1-13]. These changes contribute as ingredients to evaluating integrity of the structure. The methods using measurement of lower structural frequency changes can be used to identify, locate and estimate severity of the damages in the structures as a NDT technique. This method can be applied to any kind the structures without need to measured healthy structure data [1-2]. However, change in frequency itself may be provided information regarding the damage presented in the structure and not the location and quantification of damage. Pandey et al [3] suggested damage identification method based operational deflection shapes and its curvatures that the absolute values of differences in curvatures of mode shapes are as significant damage indicators in their numerically analyzed structural beam. In the recent years an extensive literature reviews have been published on various damage detection methods and health monitoring of structures based on vibrational dynamic parameters i.e. natural frequencies, modal displacements, and ratios of modal damping values, etc. [4-5]. Furthermore, the existence the damage in the structure not only leads to change global dynamic characteristics but also locally around the area of damage which causes to alter both global and local the strain energy of the structure. Cornwell et al. [6] utilized the experimental and numerical modal parameters to detect the damages in structural plate-like members based on the strain energy change and developed strain energy-based damage index for structures characterized by one dimensional beam and bidimensional plate –like structures. Energy based methods successfully applied to identify location and quantification the damage in a storey steel frame or truss structures by Shi et al [7-8]. To compute complex partial differential equations the differential quadrature method was used and the effectiveness of strain energy-based damage index was significantly affected by resolution of mesh grid, the visibility the location of the damage increases with finer mesh grid [9-10]. Rucevskis et al [16-17] proposed damage detection method based on mode shape curvatures in plate- like structures requiring mode shape data only from damaged structures. A node residual force vector based numerical technique to locate and estimate the degree of damage elements. Effectiveness of the method investigated by module of MatLab [25] . New damage localization method for beam like structures by using mode shape data extracted from moving vehicle response function He et al [18]. A continuous wavelet transforms technique and mode shape curvature method autoregressive used to identify damage in two carbon/epoxy composite beams with impact damage of different severities [19] . Dahak et al [20] presented a new damage structural damage identification method using frequency contour lines, obtained from various damaged conditions.

Potential damage indicator and estimator to detect and evaluate the internal surface cracks by designing the multiple orthogonal wavelet coefficients and treated the boundary effects by using the biharmonic spline interpolation [21]. Pedram et al [22] developed frequency domain-based damage detection method for the damage identification of plate and shells using power spectral density function. Three- stage algorithm proposed using a vector support machine based on several statistical parameters of damaged and undamaged noise correlation function [23]. Presented a compact rectangular phase piezoelectric transducer array-based damage detection method for plate- like structure damage detection [24]. However, change in the natural frequencies or mode shapes and its mode shape curvature from the first few operational deflections shapes of structure may not be sensitive to the effect of small cracks in the structure. Thus, an alternative to attempts to solve this problem is by using modern- techniques such as damage identification in structures based on spatial measures signal of the structure. The basic concept of this method is using the higher differentials of measured signal or dynamic spatial signal of a structure as to ascertaining specific characteristics of the structure. Therefore, in recent decades many researchers have been attracted towards the modern-type damage detection methods in structure because this method provided simple and also feasible solution to the problems. Another main advantage of this methods compared to traditional-type is having cable to identify the tiny structural damages without base line data as reference. In recent years, an applications of mode shape curvature theory were introduced to mode shape displacements for accurate characterization of damage in beam and plate-like structures. Because differential of mode shape has an ability to examine the local and global damages in spatial signal with fine focus to furnish multiple levels of detail information of the signal. Unfortunately, significant draw back of the all methods stated above is that they required dynamic characteristics of the healthy structure as a base line. It is increasing the experimental cost and consumed the huge amount of time. Thus, in the view of above consideration, this research more effort has been made to derive an approximately equivalent of healthy mode profiles from damaged mode profiles. The curve fitting (CF) analysis was used to extract the healthy modes from damaged one. The accuracy of the healthy mode profiles is validated through numerically obtained mode profiles of the damaged beam. Further, the effectiveness of the proposed method such as mode shape curvature in detect the location and quantification of the damage demonstrated by operational mode shapes extracted from numerical analysis carried on single and double cracked aluminium cantilever beam. In addition, effect higher modes on damage in structural beams also analysed.

THE PROPOSED DAMAGE INDICATOR FOR BEAM STRUCTURES

Different authors have used all the aforementioned methods in literature to predict the presence of damage and its location in the structure. Generally, all above mode shape and derivatives of mode shape such as curvature-based method is originally formulated to identify the damage in structure that is characterized by intact and damaged one-dimensional beam-like structures. In this present work, proposed structural damage detection technique based SMSC formulated to accurately predict the location and size of the crack in one-dimensional beam-like structure using accurately composed healthy mode shaped from damaged structure. First, the finite element analysis is considered to extract structural response data i.e. modal frequencies and corresponding mode shapes of damaged actual 41 grids of beam structure. Resulting this, one- dimensional transverse displacement matrices are obtained for corresponding to kth mode shape of damaged beam structure as shown in Eq (1). Similarly, Eq (2) represents mode shape associated with healthy structure is obtained by using curve fitting analysis.

In this method requires only few structural mode shapes after damage.

Step1: Calculating mode shapes of beam structure

$$\sum_{j=1}^{n}\Psi_{k,j}{}^{d} = \Psi_{k,1}{}^{d},\Psi_{k,2}{}^{d},........\Psi_{k,(1,n)}{}^{d} \quad (1)$$

$$\sum_{j=1}^{n}\Psi_{k,j}{}^{h} = \Psi_{k,1}{}^{h},\Psi_{k,2}{}^{h},......\ \Psi_{k,(1,n)}{}^{h} \quad (2)$$

Where, d k , j and h k , j are transverse displacements of jth (j = 1,2, … . . , n) measured grid or node of kth mode shape of the beam-structure as shown in Figure.1, whereas, “d” and “h” are denote damaged structures, respectively.

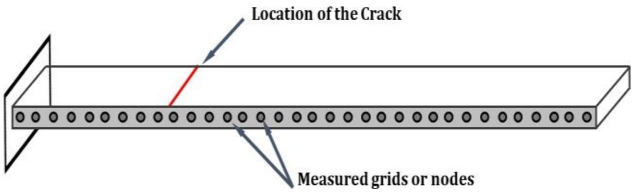


FIGURE 1: Basic beam structure with node or measured grid

Step 2. Generating mode shape curvatures

Then, with one-dimensional transverse displacement vector, mode shape curvature at jth measured grid or node corresponding to kth mode of damaged can be calculated using the central finite difference approximation. The resulting equations for damaged and intact mode shape curvatures are presented in Eqs (3) and (4).

$$\Psi''_{k,j}{}^{h} \quad \frac{\left|\Psi_{k,j-1}{}^{h} - 2\Psi_{k,j}{}^{h} + \Psi_{k,j+1}{}^{h}\right|}{l^2} \qquad (3)$$

(4)

$$\Psi''_{k,j}{}^{d} \quad \frac{\left|\Psi_{k,j-1}{}^{d} - 2\Psi_{k,j}{}^{d} + \Psi_{k,j+1}{}^{d}\right|}{l^2}$$

Where, , l is distance between two adjacent grid points or nodes (length of the element) along x and y directions, respectively.

Step3: SMSC based damage index

Then, Square of Mode Shape Curvature (SMSC) based damage indices DIk , j are computed along x and y directions with help of Eq (5). Damage index can be defined as the absolute difference between the square of mode shape curvature damaged and undamaged plate structure either x or y direction.

$$DI = \sum\sum \left(\Psi''\right)^2 - \left(\Psi''^{\,h}\right) \Big| \quad (5)$$

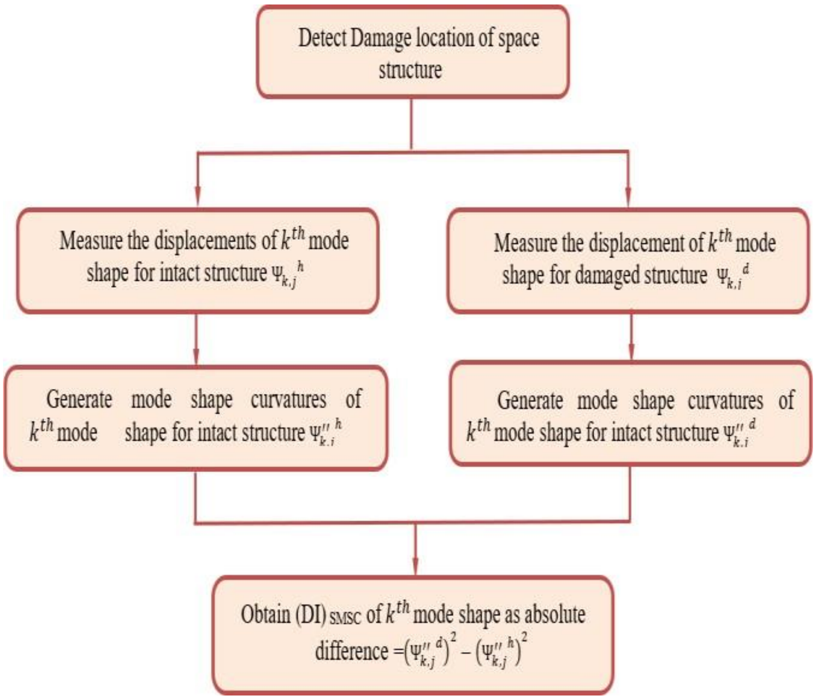


FIGURE 2. Damage diagnosis methodology

NUMERICAL ANALYSIS OF A CANTILEVER BEAM

Numerical analysis is performed using commercially available FEA software i.e. ANSYS 15.0. Solid element (Brick 8 node 185) is used to model an aluminium cantilever structural beam having dimensions i.e. width B =0.025m, height H= 0.001 and length L =0.75 m, with hexagonal mapped meshing of elements. The boundary conditions used in Finite Element Modal Analysis (FEMA) are as same as in EMA. Figure 14 shows that frequency spectrum of first five vibration modes of healthy structural beam. The comparison of first five natural frequencies obtained in EMA and FEMA are shown in Table 2. Damage has simulated as rectangular open transverse crack by reducing the stiffness of elements 10%, 20% and 30% of the beam height at location of damage. Material properties of the beam aluminium beam 6061 is given in the Table.1.

TABLE 1. Material properties of double cracked beam

Name of the property	Magnitude	Measured units
Young’s modulus	$70\text{x}10^9$	N/m^2
Density	2700	Kg/m^3
Poison ratio	0.3	----

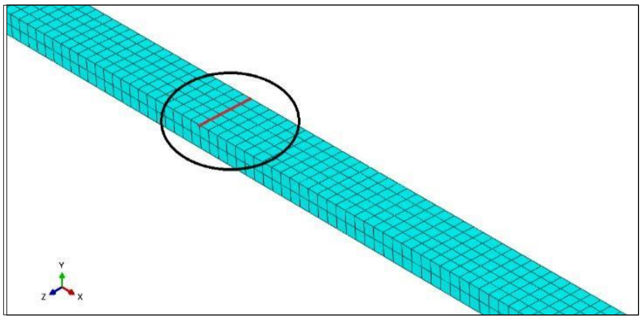


FIGURE 3. Schematic Diagram of simulated single cracked Beam: Damage location at 0.5L

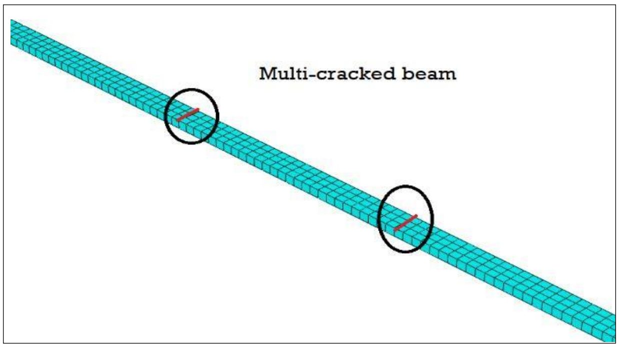


FIGURE 4. Schematic Diagram of simulated Double cracked Beam: Damage location at 0.25L and 0.75L.

EXPERIMENTAL MODAL ANALYSIS (EMA) ON STRUCTURAL BEAM

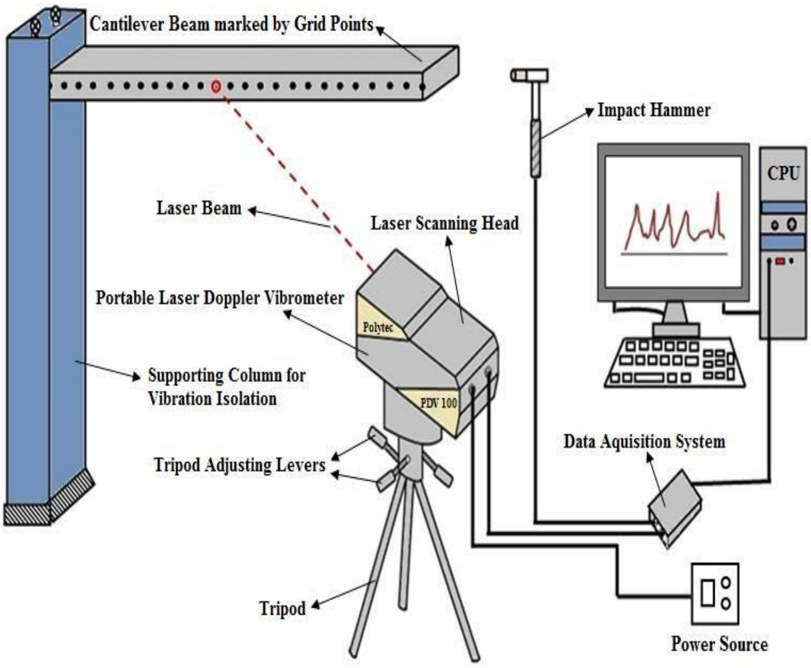


FIGURE 5. Schematic layout of Experimental set-up

Experimental Modal Analysis (EMA) is conducted for damage detection in structural beam using Portable Digital Vibrometer (PDV-100), non- contact vibration measuring equipment. General experimental set-up consists of 3-D Polytec PDV-100 optical scanning head with wide frequency range 0 to 22kHz, highly sensitive vibrometer sensor (OFV- 550), VIB-E-220 junction box, OFV-5000 Modular Vibrometer controller, and a computer system withVibSoft-20 data acquisition and analysis software. The detailed experimental set-up is shown in Figure 5. A structural beam of aluminium 6061 with the dimensions 0.75 × 0.025 × .01 mm3 is used in EMA. One end of the structural beam is clamped to replicate cantilever (fixed-free) boundary conditions as shown in Fig. 5. The beam is marked by 41 grid points along length wise with the element length of 0.02 m. Modal displacement data is collected from the measurements, which are further utilized to derive mode shape curvatures using central-difference approximation method. Two types of damage scenarios of beam are taken for the experimentation: single surface open crack at mid span of beam and two surface open cracks at 0.2 m, 0.6 m locations along beam's length. The Schematic representation of damaged beams shown in Figure 6 and 7. The cracks are formed using Electron Discharge Machining (EDM) with a depth of 1 mm in both the damaged cases. Same specifications of damages are taken for FEMA. The accuracy of proposed methodology in surface characterization has been validated by an experimental data acquired by a non-contact measurement (scanning laser vibrometer with impact hammer). One of the main advantages of the Laser vibrometer is that it prevents mass loading of the test object, resulting in more accurate results than using traditional contact vibration sensors such as accelerometers, which can change resonance frequencies and damping values. Second advantage is the possibility to record the response of individual locations or at many locations in rather simple since it is not necessary to install a transducer on every measured point and structures of nearly any size can be studied.

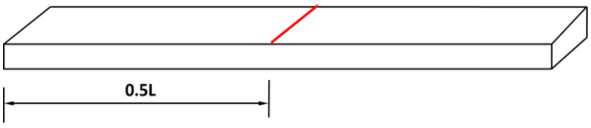


FIGURE 6. Location of single crack in the beam

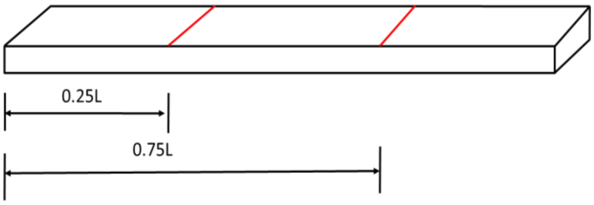


FIGURE 7. Location of Double cracks in the beam

RESULTS AND DISCUSSIONS

In the present research work, mode shape-based damage identification method such as curvature mode shape approach is investigated using the composed undamaged mode profiles as a base line data. To assess the effectiveness of proposed approach, numerical modal analysis is employed on cantilever beam considering single and multi-damage location with various crack depth such as 10%, 20% and 30% height of the structural beam, experimental modal analysis on aluminum beam with Fixed-Free boundary conditions is also carried out to balance with the numerical results. First five the natural frequencies and mode shapes of both numerical and experimental data with considering single and double cracked listed in Table 1. It is observed that change in frequency of first few fundamental frequencies almost negligible in this work, which indicates that compared to low frequency modes higher modes are more sensitive to damage [16-17]. In order to validate and evaluate the performance of the proposed approach, results have been compared with an effective previously proposed method that was verified based on experimental results [12].

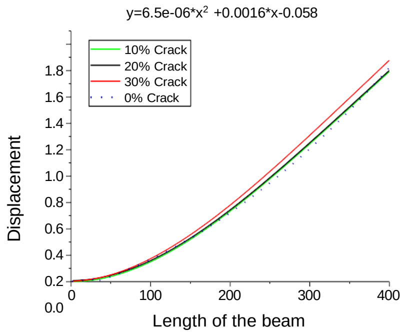


Figure 8: First five numerical displacement modes shapes

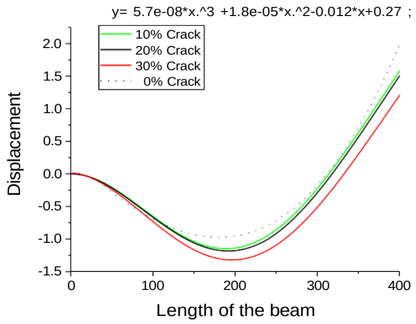


Figure 9: First Mode profiles of healthy and cracked beam

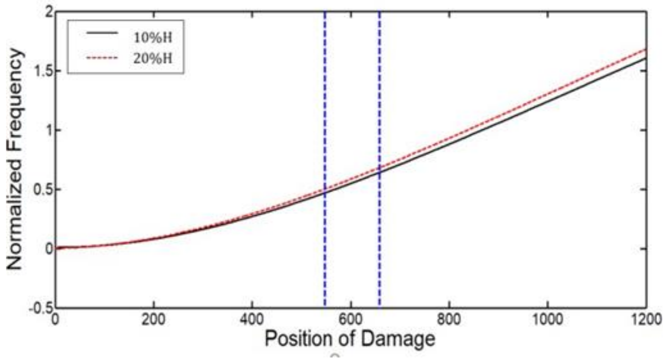
All above methods discussed in the literature are ascertaining location of the damage by data calculated absolute difference between the mode shapes of structure before and after the damage. To compute the data of healthy state of structure is laborious work or sometimes even impossible. To overcome this problem, curve fitting (CF) analysis is employed to obtain mode profiles of undamaged structure. The basic idea of this originated from damaged mode profiles, which are not showing any physical significant change in their deflected shapes before and after the damage of the structure. Figures 8 and 9 shows that operational mode shapes of mode1 and mode2 of cracked and un-cracked beam, dotted line indicates mode profile of healthy state of structure which is generated by using curve fitting analysis in MatlabR2015a, and cracked beam with varying crack depth from 0.1h to 0.3h are similar and even though crack depth increases mode shapes not exhibits any physical change in their mode profiles of different crack depths. Hence, damage cannot identify just looking at the mode profile itself. Alternative solution to attempt this problem is curvature mode shapes, which are more sensitive to damage is presented in next section.

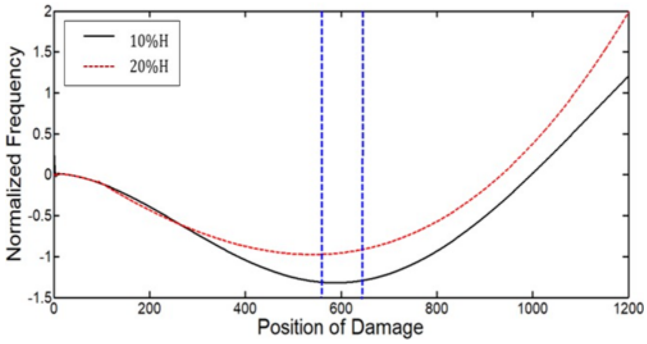
Curvature mode shape-based Damage identification method

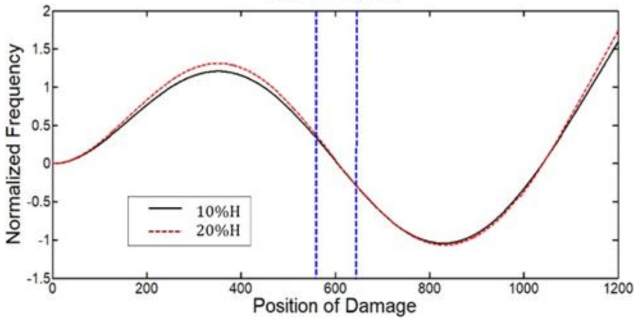
In this section, mode shape curvature-based damage (CMS) identification method is presented, curvature is the derivative of the operational deflection shape, which is more sensitive to damage to identify the location of the damage [12]. To illustrate characteristics the proposed damage identification method the beam with different scenarios in terms of single, multi-crack and single crack with various crack depths are investigated. From Figure. 10, it can be seen that the damage index obtained from CMS is successfully identified the location and quantification of single and multi-damages in beam. Damage index obtained from noise free mode shape clearly identified the location of crack by showing large peaks at the crack location. Furthermore, the effectiveness of proposed method also evaluated in quantify damage by varying the crack depth from 0.1h to 0.3H, which succeeded in efficiently estimate the damage location and quantification in beam structure as shown

Case study 1: Damage identification and quantification in Single cracked beam.

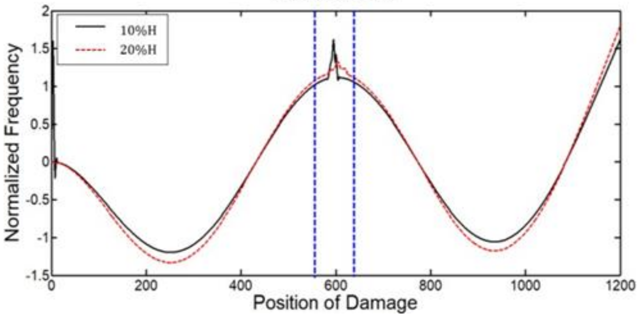
Results about position of single crack presented by showing distribution of damage indexes of proposed method along the length of the beam structure. This method clearly identifies the location single crack, which simulated at the center of the beam structure as shown in Figure. 10. In order to estimate the effectiveness of the proposed method in quantifying the damage, the beam was simulated with different crack depths such as 10%, 20% and 30% height of the beam. From the plot Figure.11, one can be observed that this method is succeeded in identify the severity of the damage by showing the different damage indexes such as 2e13, 5e13 and 7e13 for the different crack depths 0.1H, 0.2H and 0.3H. It can be seen that the damage index obtained from the mode shape of beam with crack location at 0.5L and depth 0.3h indicates the highest damage indices at location of crack compared to other damage indices obtained from relative crack depths such as 20% and 10% height of the beam. This happen due to the loss of stiffness is higher with increase in crack depth.


(a1)-Mode1(FEA)


(a2)-Mode 2 (FEA)


(a3)-Mode 3 (FEA)

(a4)-Mode4 (FEA)



(a5)-Mode5(FEA)

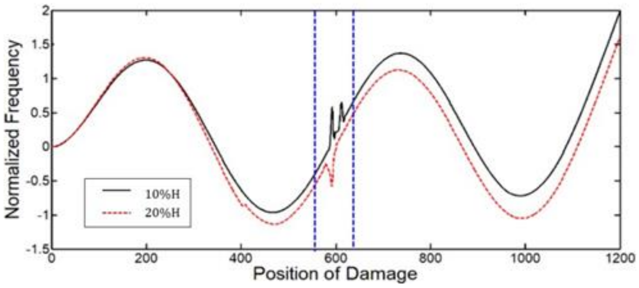
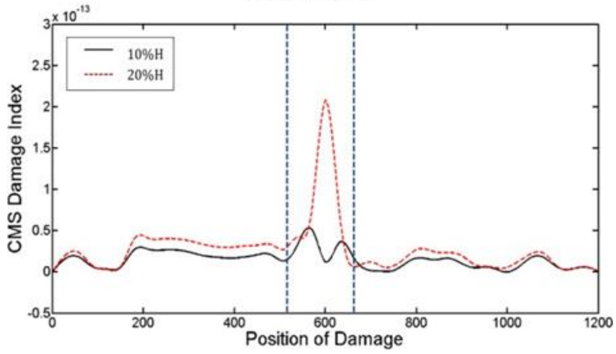
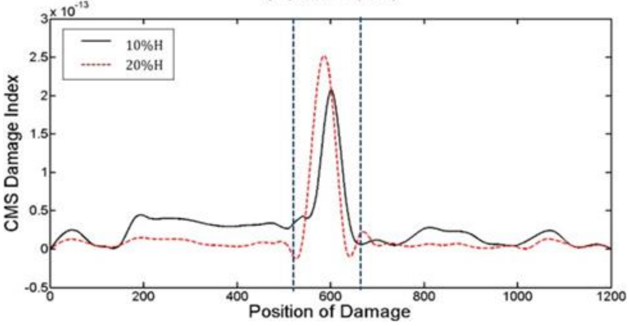


FIGURE 10. (a) Numerical and Experimental mode shapes

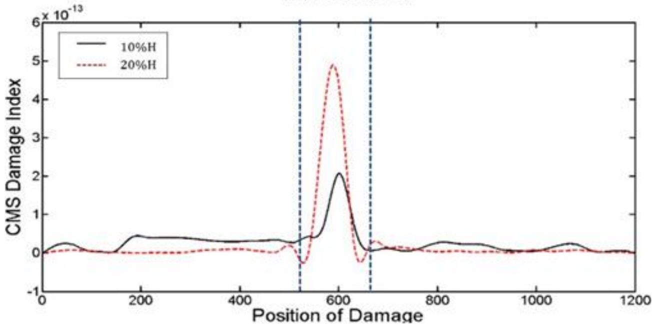
(b1)-Mode1(FEA)



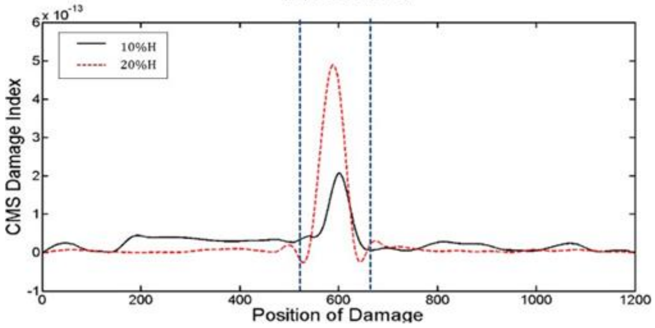
(b2)-Mode 2 (FEA)



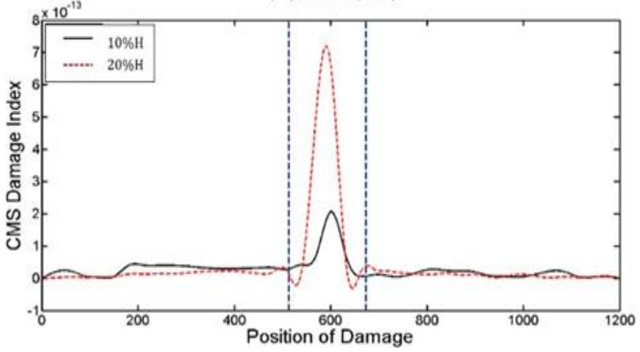
(b3)-Mode 3 (FEA)



(b3)-Mode 3 (FEA)



(b4)-Mode 4 (FEA)



(b5)-Mode 5 (FEA)

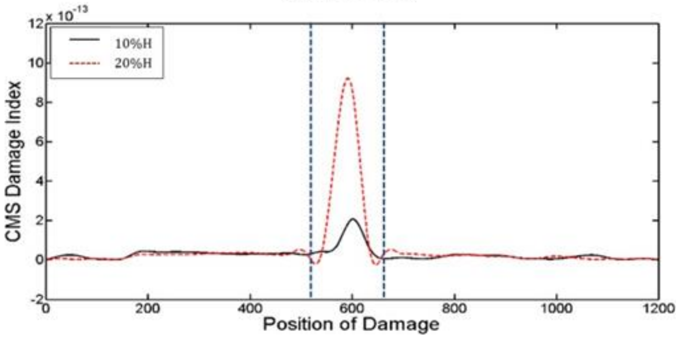


FIGURE 10. (b) Numerical and experimental damage indicators with with single crack single crack at0.5L

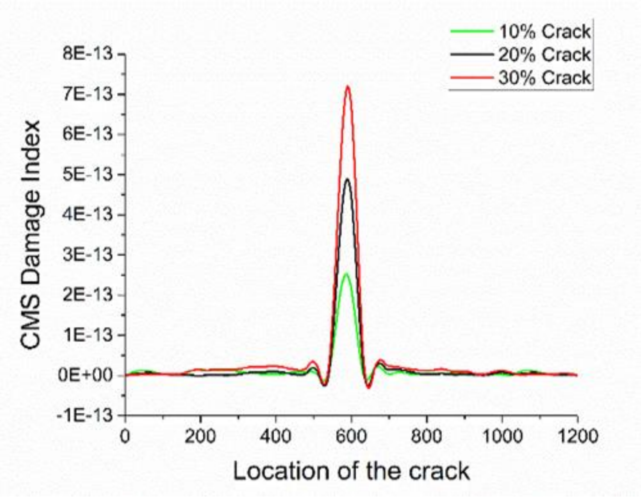


FIGURE 10. Single cracked cantilever beam (crack location at depth 0.5L; crack depth from 0.1H to 0.3H

Case study 2: Influence of mode order towards the damage

Further, the effect of higher modes on damage is investigated by using the first five numerical mode profiles. The damage index is computed from higher modes are showing highest peaks at location of the damage compared to damage indexes corresponding to lower mode damage indices. From Fig. 12, it can be concluded that higher modes are more sensitive to existence of the damage which means that modulus of the damage index is increases with higher mode order [17].

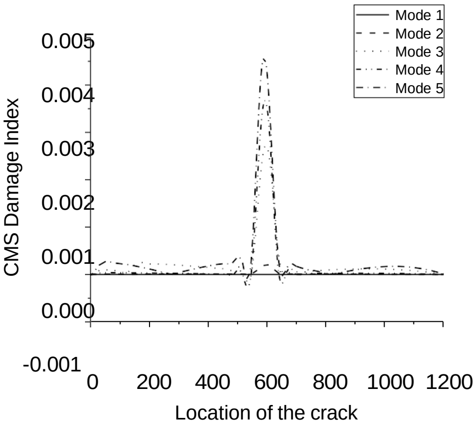


Figure 12: Effect higher modes to damage in the beam (crack location at depth 0.5L)

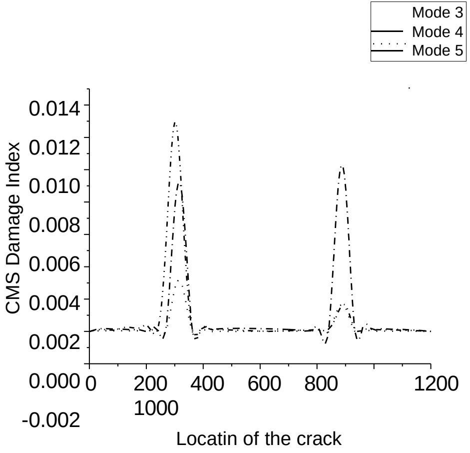


Figure 13: Double cracked cantilever beam with crack locations at 0.25L and 0.75L; depth 0.1H

Case study 3: Damage identification and quantification in Double cracked beam

In this section, aluminum cantilever beam with multiple crack locations are studied. For illustration the proposed method a cantilever beam model is simulated in the Ansys15.0 to extract the mode profile information with considered Clapped-Free (CF) boundary conditions. Geometrical dimension and appropriate crack locations are given in the Figure 7. The dimensions of double cracked beam as follows: length L = 750 mm, height H= 10 mm and width B=25mm. Two cracks in the beam are simulated in same way as in case of single cracked beam such as depth 2mm and width 1 mm, located at 200mm measured from each end of the beam structure. The beam uniformly divided into 400 elements with each element length is 2mm. Material properties of double cracked aluminum cantilever beam are selected same as for beam with a single cracked beam as given in the Table 1. Again, first five natural frequencies and corresponding operation deflection shapes for different damage scenarios in terms of multi-crack with 1mm width and increase in crack depth 0.1H to 0.3H are extracted for the analysis. This study also helps full to investigation damage in different locations. Figure 13 which reveals that damaged elements near the boundary are more influenced by the damage relatively damage elements which are away from the fixed boundary and damage indices obtained mode3, mode4, and mode 5 exhibits highest peak values at the location of the crack. But this method is failed to identify the location of the crack in case of double cracked beam with first and second mode shape, this is happened may be insufficient quality of the signal. Otherwise, peak value of damage indices of lower modes is invisible in front of peak values of higher modes, as show in Figure 12. Further, this method employed on fourth mode profile to identify the severity of damage in double cracked cantilever beam with different crack depths such as 10%, 20% and 30% height of the beam and cracked at 0.25L and 0.75L, corresponding results plot in Figure 13.

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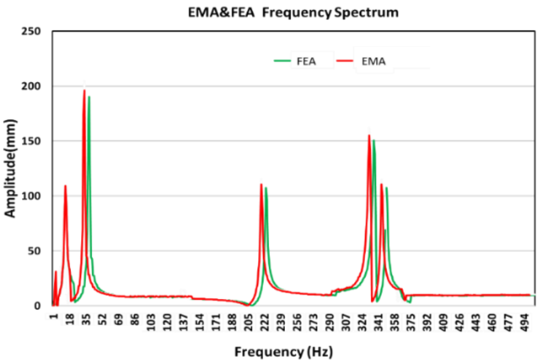
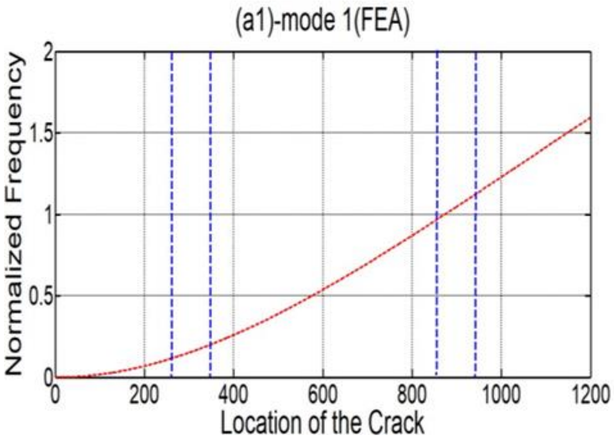
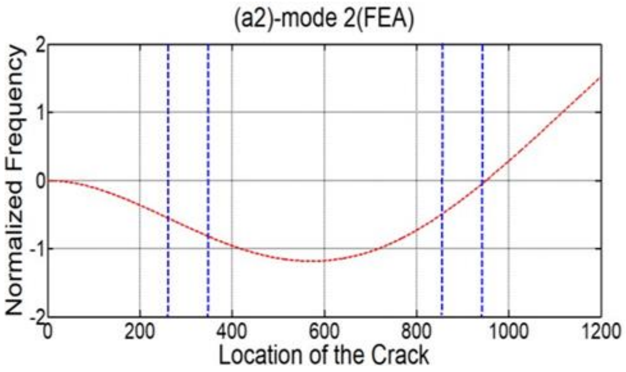
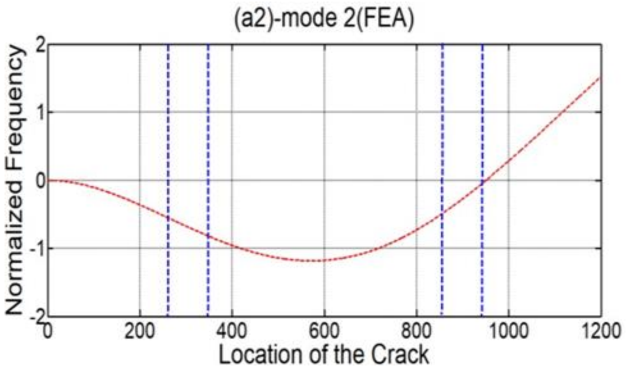
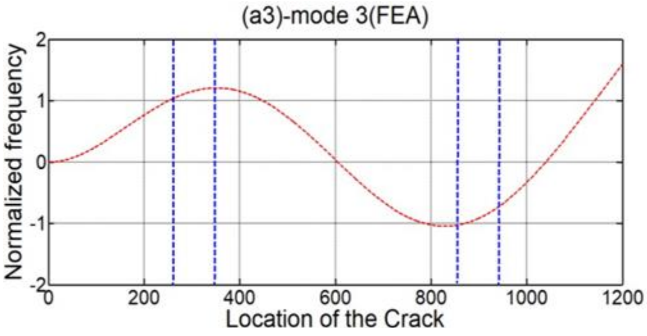


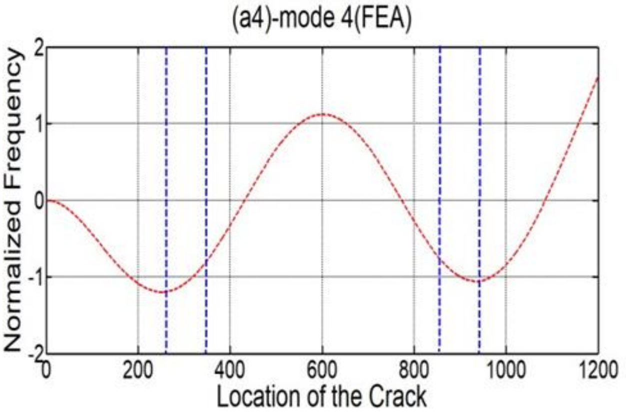
FIGURE 14. Frequency spectrum of healthy structure from FEMA and EMS.




(a2)-mode 2(FEA)
Normalized Frequency
2
1
0
-1
-2
0
200
400
600
800
1000
1200
Location of the Crack


(a2)-mode 2(FEA)
Normalized Frequency
2
1
0
-1
-2
0
200
400
600
800
1000
1200
Location of the Crack


(a3)-mode 3(FEA)
Normalized frequency
2
1
0
-1
-2
0
200
400
600
800
1000
1200
Location of the Crack


(a4)-mode 4(FEA)
Normalized Frequency
2
1
0
-1
-2
0
200
400
600
800
1000
1200
Location of the Crack

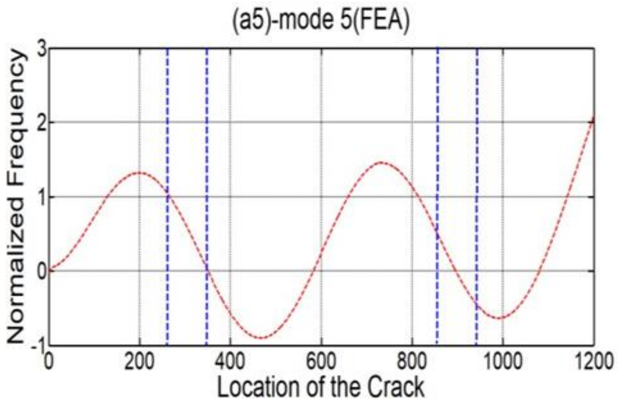
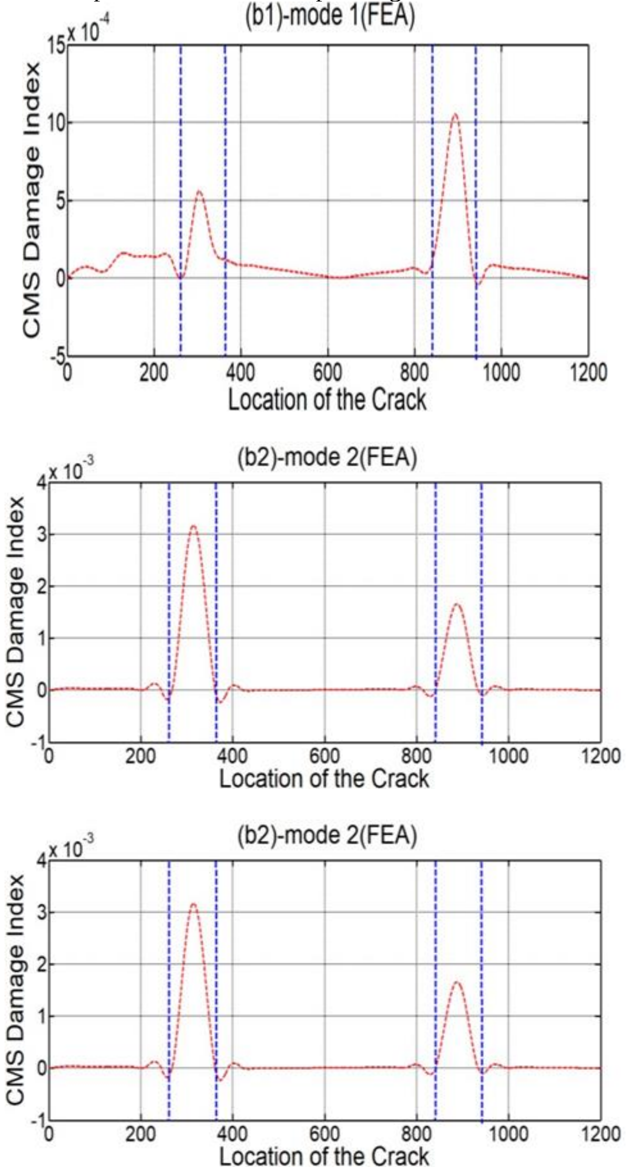


FIGURE 15. (a) Numerical and Experimental mode shapes single crack



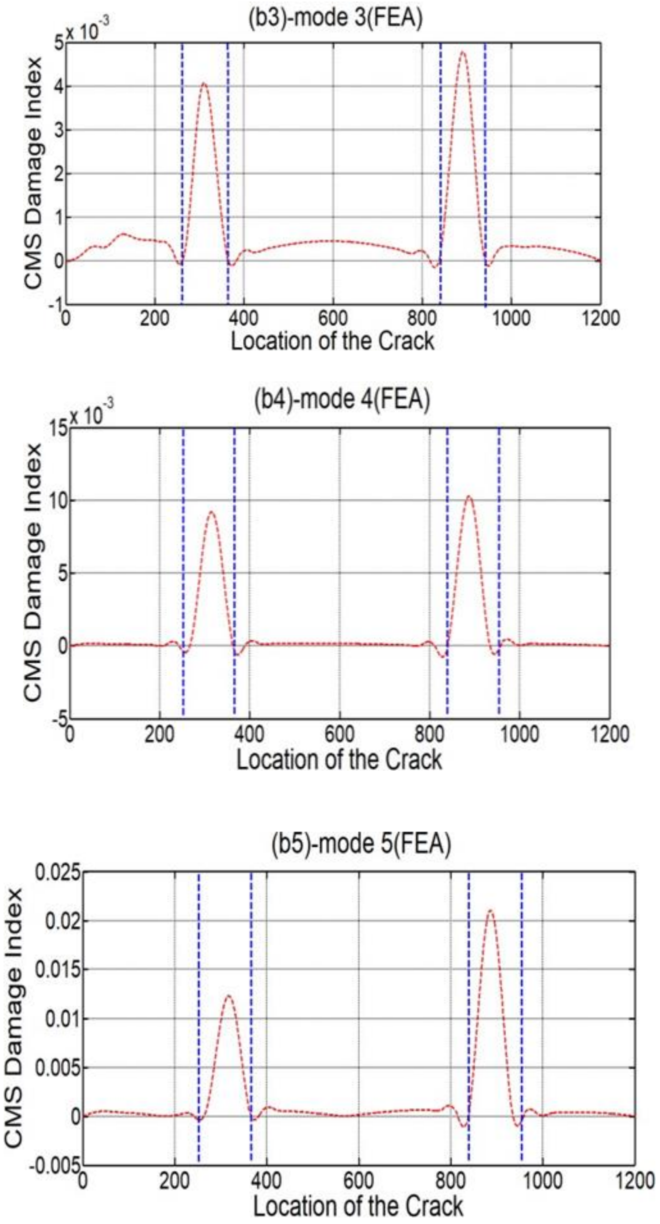


FIGURE 15. (b) Numerical and experimental damage indicators with single crack at with at 600mm   600mm

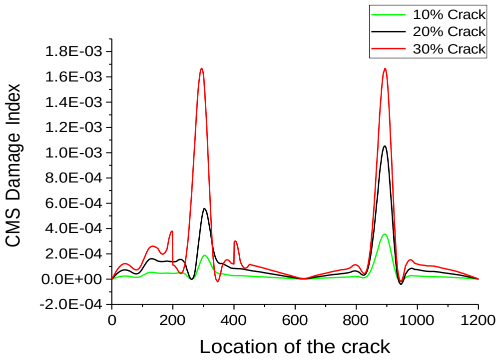


FIGURE 16. Double cracked cantilever beam (crack location at depth 0.25L and 0.75L; crack depth from

TABLE 1. Natural Frequencies of different damage cases

	Before Damage	After Damage (single	After Damage (multi-crack)

Mode. No. (*i*)			crack)			
	EMA (Hz)	FEMA (Hz)	EMA (Hz)	FEMA (Hz)	EMA (Hz)	FEMA (Hz)
1.	16.2500	17.6250	15.6250	16.258	15.1250	15.1119
2.	34.2560	36.8750	31.2500	35.3040	30.5200	31.2731
3.	225.0000	230.2215	215.2250	227.2980	214.8750	224.2126
4.	332.0000	335.1240	328.3600	332.1200	325.0000	321.3742
5.	348.7500	353.4500	339.1150	349.6900	336.1500	345.1318

CONCLUSION

This work mainly focuses on identification of location and quantification of the damage in single and double-cracked aluminium cantilever beam by using accurately composed undamaged mode profiles and measured damaged ones. Dynamic parameter of vibrational structural beam obtained from numerical and experimental modal analysis. It was proposed to use the curvature mode shape method with obtained mode profiles as base line data and compared results with experimental results. The effectiveness of the proposed methods is demonstrated by first five modes extracted from two aluminium cantilever beams containing single and double-cracks at different locations and depths. The results of research lead to following conclusion: The damage index obtained based on curvature mode shape required the dynamic parameters of the vibrational structure before and after the damage to locate and quantify the damage. To reduce the time and experimental cost, mode shapes derived by the proposed approach can be conveniently used as reference data for the damage identification and estimation purpose. Further, proposed method is also performed to detect the damage in a single and multi-cracked beam. The main advantage of CMS based damage index is that this method has capability to identify the damage in the structure even with lower mode shape itself. The apparent limitation of CWT is that which requires mode shape measured with a relatively high spatial resolution. This was overcome by enhanced the signal using the cubic spline interpolation technique. For the established damage index values, for higher mode profiles, the peak value of damage index is also higher, which implies that higher mode profiles are more sensitive to the presence of the damage. Worthy of noticeable point in the damage detection methods is that are requiring healthy data as the reference. In general, the baseline data may not be available or even impossible to obtain. Although, curvature method is performed based on base line data, which derived artificially from damaged mode profiles.

FUTURE WORK

Further, study towards the application of the proposed approach to two- dimensional plate-like structures seem worthy of investigation. These related problems are intended to address in future work.

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