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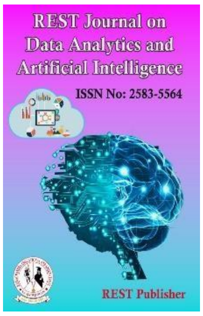

Dominating Sets in Fuzzy Graph

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Abstract. *In this paper we focus on Dominating set, Minimum Dominating set, Fuzzy graph and Dominating number in Fuzzy graph. Using this new concept on domination in fuzzy graph some important theorems has been proved this paper. The independent domination is also defined with this concept. The domination number (γ), the independent domination number (i) and the bounds on these parameters are discussed.*

1. INTRODUCTION

The concept of domination and determines the domination number for several fuzzy graphs. In this paper we introduced the concept of domination in fuzzy graph and dominating number in fuzzy graph. And also we discussed about independence set, independent domination number and bounds on the domination number. Also several applications in problem life were solved by these branches. So study of domination in fuzzy graphs worth to care.In this work we introduce the concept of fuzzy domination number in fuzzy graph by effective edge.

DEFINITIONS 1

A Graph G comprises a set V of vertices and a set E of edges. Each edge in E is a pair (V,E) of vertices in V .If (V,E) is an edge in E,we connect V and E in the graph of drawing of G
Definition: The edge $e = (x,)$in a fuzzy graph $GF = (\eta,\rho)$ is called effective edge if $\rho\ (x,y) \leq \eta\ (x) \wedge \eta\ (y)$.
Definition: The open neighborhood of a vertex $x \in\ (GF)$ is denoted by (x) and define as $N(x) = \{y \in V/(x,y)\ iseffectiveedge\}$and $N[x] = N(x) \cup \{x\}$ is called closed neighborhood of a vertex x.
Definition: Afuzzy graph $GF = (\eta,)$is called strong if each edge belongs to it is effective.
Definition: The degree of a vertex is the number of effective edges. And denoted by deg (x). The minimum degree of $GF$is $(GF) = \wedge\ \{(x),x \in V\}$ and the maximum degree of $GF$is $\Delta(GF) = \vee\{d(x)/\ x \in V\}$.
Let V be a finite non empty set and E be the collection of all two element subsets of V. A fuzzy graph G(σ,) is a set with a pair of relations σ:V→ [0, 1] and : VXV → [0, 1]
Such that (u, v) ≤ σ(u) σ(v) for all u, v in V. The scalar cardinality of a fuzzy set V is the sum of the membership grades of the elements of the fuzzy set and That is $\sum\sigma(V)$. The order and size of a fuzzy graph is defined by the scalar cardinality of V and the scalar cardinality of VXV and are denoted by 'p' and 'q' respectively. That is = p and = q. Let G (σ,) be a fuzzy graph on V and V1 V. Define 1 on V1 by 1(u) = (u) for all u V1and define 1 on V1XV1 by (u,v) = (u,v) for all u, v V1. Then G1 (1, 1) is called a fuzzy sub graph of G induced by V1 and is denoted by <V1>. is called the complement of a fuzzy graph G(σ,), where . An edge (u, v) =e is called an effective edge if (u, v) = σ(u) σ(v). If (u, v) is an effective edge, then u and v are adjacent effective edges. A fuzzy graph is said to be strong fuzzy graph if (u, v) = σ(u) σ(v) for all u, v in V. A vertex u is said to be isolated vertex (u, v) < σ(u) σ(v) for all v V-{u}. A fuzzy graph G(σ,) is said to be complete fuzzy graph, if all the edges are effective between every pair of vertices and is denoted by N(u)={v V/ (u, v) = σ(u) σ(v) } is called the open neighborhood of u and N[u] = N(u) {u} is the closed neighborhood of u. If an edge (u, v) is an effective edge, then its incident is said to be effective incident with both the vertices. The effective incident degree of a vertex u of a fuzzy graph is defined to be the sum of the scalar cardinality of the effective edges incident at u and is denoted

by dE(u). The minimum effective incident degree is E(G) = min {dE(u) / u V} and the maximum effective incident degree is E(G) = max {dE(u) / u V}. A fuzzy graph G is said to be bipartite if the vertex set V can be partitioned into two sets V1and V2 such that (v1,v2
)=0 if (v1,v2 V1 X V1 or (v1,v2) V2 X V2 Further if (u, v) = σ(u) σ(v) for all u V1 and v V2, then G(σ,) is called complete bipartite fuzzy graph and is denoted by . Hereafter we write G for G(σ,).

2. DOMINATION IN FUZZY GRAPH DEFINITION

Let G(σ,μ) be a fuzzy graph on V and let u, v V. If μ (u, v)=σ(u) σ(v)= σ(v) then u dominates v (or v is dominated by u) in G. A subset D of V is called a dominating set in G if for every v -D then there exist u D such that u dominates v. A dominating set S of a fuzzy graph G is said to be a minimal dominating set (MDS) if there is no dominating set S' of G such that A dominating set D of a fuzzy graph G is said to be a minimum dominating set, if there is no dominating set D' of G such that The minimum fuzzy cardinality of a minimum dominating set of G is called the domination number of G and is denoted by (G) or γ

Example 3.1: Consider the fuzzy graph G(σ,), where ={v1/0.1, v2/0.5, v3/0.8}, and v1, v2)/ 0.4, v2, v3)/ 0.5}. {v1, v3} is a dominating set but {v2} is not a dominating set, because v2 dominates v1 but not dominates v3.

Remark 3.1: For any u, v V, if u dominates v need not imply v dominate u. For any u, v V, if μ (u, v) = σ (u) σ(v) = σ(u) = σ(v) then u dominates v and also v dominates u. For any u V, N (u) is precisely the set of all v V which is adjacent to u. If μ (u, v) < σ (u) σ(v) for all u, v V then clearly the only dominating set in G is V. Theorem 3.1: For any fuzzy graph G. $\gamma + \bar{\gamma}$ = 2p if and only if 0 < μ (u, v) < σ (u) σ (v) for all u, v V where is the domination number of .

Proof: From the definition of isolated vertex, μ (u, v) < σ (u) σ (v) for all u, v V and = p μ (u, v) - σ (u) σ (v)< σ (u) σ (v) for all u, v ϵV. Hence = 2p if and only if 0 < μ (u, v) < σ (u) σ (v) for all u, v ϵ V. Remark 3.2: For any fuzzy graph, 2p.

Theorem 3.2: A dominating set D is a minimal dominating set (MDS) if and only if for each d one of the following two conditions holds d is not adjacent to any vertex in D (i) For some u D: (i) (or) (ii) there exists a vertex c V-D such that N(c) D = {u}.

Proof: Assume that D is an MDS of G. Then for every vertex d is not a dominating set. This means that some vertex c in (V-D) {d} is not dominated by any vertex in D-{d}. Then there are two cases.

Case (i): If c = d in which case d is adjacent to any vertex u in D such that .

Case (ii): If c V-D then c is not dominated by any vertex in D-{d}, but it is dominated by some vertex u in D then u is adjacent to only vertex c in V-D. Thus N(c) D = {u}.

Conversely, suppose that D is a dominating set and for each vertex d one of the two conditions holds. We now show that D is an MDS. Suppose that D is not an MDS, and then there exists a vertex d such that D-{d} is a dominating set. Hence d is adjacent to at least one vertex in D-{d}, and then condition (a) does not hold. Also if D-{d} is a dominating set then every vertex of V-D is adjacent to at least one vertex in D-{d} that is conditions (i) or (ii) does not hold. This is a contradiction to our assumption that at least one of the condition holds.

Remark 3.3: Let G be a fuzzy graph without isolated vertices and let D be a dominating set of G. Then V-D need not be a dominating set of G. If all the vertices having the same membership grade then V-D is also a dominating set.

Examples 3.3*:* Consider the fuzzy graph G(σ,), where ={v1/0.8, v2/0.2}, and v1, v2)/ 0.2}. D = {v1} is a dominating set of G but V-D = {v2} is not a dominating set of G. Consider the fuzzy graph G(σ,), Where ={v1/0.8, v2/0.8}, and v1, v2)/0.8}. Then D={v1} and V-D={v2} are the dominating sets of G.

3. INDEPENDENT DOMINATION IN FUZZY GRAPH

Definition 4.1: A set S in a fuzzy graph G is said to be independent if μ(u,v) < σ(u) for all u, v S.

Definition 4.2: A Dominating set is called an independent dominating set if D is independent. An independent dominating set S of a fuzzy graph G is said to be a maximal independent dominating set (MIDS) if there is no independent dominating set S' of G such that An independent dominating set S of a fuzzy graph G is said to be a maximum independent dominating set if there is no independent dominating set S' of G such that The minimum scalar cardinality of an maximum independent dominating set G is called the independent domination number of G and is denoted by (G)

Example 4.1: Consider the fuzzy graph G(σ,), where ={v1/0.2, v2/0.3 v3/0.6, v4/0.8 v5/0.4, v6/0.1} and v1, v3)/ 0.2, v2, v3)/ 0.3, v3, v4)/ 0.6, v4, v5)/ 0.4, v4, v6)/ 0.1}. S1={v3,v5,v6}, S2 = {v1,v2,v4} and S3={v1,v2,v5,v6} are the independent dominating sets. The independent domination number $i(G) = 1.1$ Remark 4.1: For any fuzzy graph G, (G) (G).

Definition 4.3: The independent domination number (G) of a fuzzy graph G equals the minimum scalar cardinality of the set in MIDS. The upper domination number I(G) of a fuzzy graph G equals the maximum scalar cardinality of a set in MIDS.

Example 4.2: From the fuzzy graph given in the example 4.1, $i(G)=1.1$ and $I(G)=1.3$.

Theorem 4.1: An independent set is a maximal independent set in a fuzzy graph and the membership grades of all vertices are equal if and only if it is an independent dominating set.

Proof: Every maximal independent set is an independent dominating set because all the vertices having the membership grades are equal. Conversely, Let S be an independent dominating set. Now we have to show that it is maximal independent set. Suppose S is not a maximal independent set then there exists a vertex u V-S Such that S {u} is an independent set. But if S {u} is an independent set, then no vertex is adjacent to u. Thus S is not a dominating set. This is a contradiction. Hence S is a maximal independent set. ***Theorem 4.2****:* Every maximal independent set in a fuzzy graph G need not be a dominating set.

Proof: Let S be a maximum independent set. We show that S need not be a dominating set. Suppose the membership grades of the vertices are different, then some of the vertex u V-S are adjacent to some of the vertices v S and .

Thus, S is not a dominating set. Example 4.3: Consider the fuzzy graph G(σ,), where ={v1/0.4, v2/0.8, v3/0.5}, and {(v1, v2)/ 0.4, v2, v3)/ 0.5}. I1={v2}and I2={v1,v3} are independent sets. I1 is a dominating set but I2 is not a dominating set. Here I2 is a maximum independent set but not a dominating set.

Remark 4.2: Every maximum independent set in a fuzzy graph G is a dominating set, if all the vertices having the same membership values.

4. CONCLUSION

In this research paper a new type of fuzzy domination of fuzzy graph is presented, which it is called the closed fuzzy domination, and we have studied some basic properties and determined this type of fuzzy domination for some fuzzy certain graphs. Adding a closed fuzzy domination for some operation in fuzzy graphs discussed.

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