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Applications of Edge Coloring in Fuzzy Graphs

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Abstract. *In this paper we introduce the applications of edge coloring in Fuzzy Graph Graph coloring is an effective technique to solve many practical as well as theoretical challenges are effectively solved by graph coloring. In this paper, we have presented applications of graph theory especially graph coloring in team-building problems, scheduling problems, and network analysis.*

1. INTRODUCTION

Subjects like, electrical engineering, medicine, network analysis, operations research, and physics are the main areas graph theory has its application. With the help of graph coloring concepts, we can solve various, team building problems, scheduling problems, job allocation problems, timetabling problems etc. Graph coloring plays a immense role in Register Allocation, Map coloring, Bipartite colour checking, Mobile radio frequency assignment, Making time table. When coloring a map – or any other drawing consisting of distinct regions – adjacent countries cannot have the same colour.

Vertices: A vertex (or node) of a graph is one of the objects that are connected together. The connections between the vertices are called edges or links.

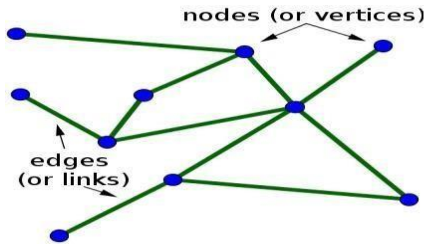

FIGURE 1. Vertices

Edge*:* An edge (or link) of a network (or graph) is one of the connections between the nodes (or vertices) of the network. Edges can be directed, meaning they point from one node to the next, as illustrated by the arrows in the first figure below. Edges can also be undirected, in which case they are bidirectional, as illustrated by the lines in the second figure, below.

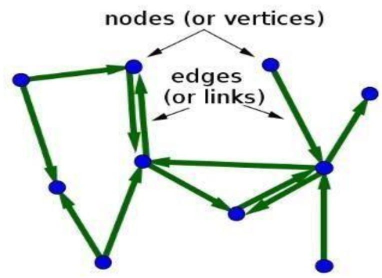

FIGURE 2. Edge https://mathinsight.org/image/small_undirected_network_labeled

Incident: If two vertices in a graph are connected by an edge, we say the vertices are adjacent. Ifa vertex v is an endpoint of edge e, we say they are incident.

Adjacent: Two vertices are said to be adjacent, if there is an edge between the two vertices. Here, the adjacency of vertices is maintained by the single edge that is connecting those two vertices. In a graph, two edges are said to be adjacent, if there is a common vertex between the two edges

Graph: A graph G consists of a pair (V(G), X(G)) , where V(G) is a non – empty finite set whose elements are called points or vertices and X(G) is another set of unordered pairs of distinct elements of V(G). The elements of X(G) are called lines or edges of the graph. If $x = \{u, v\} \in X$ then the line x is said to join of u and v. The points u and v are said to adjacent if x = u v. We say that the points u and the line x are incident with each other. If two distinct lines x and y are incident with a common point then they are called adjacent lines. A graph with p points and q lines is called an adjacent graph.

Note: When there is no possibility of confusion we write V (G) = V and X (G) = X. ***Example:***

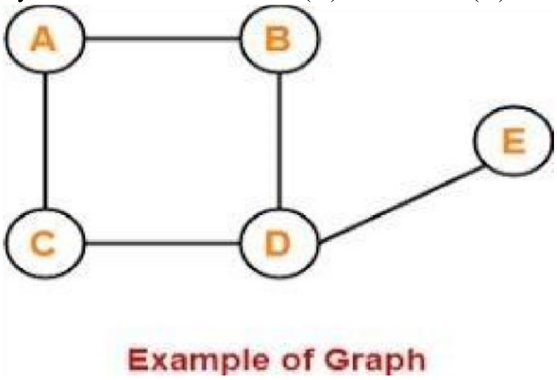

FIGURE 3. Example of Graph

Regular graph: For any graph G, we define.

(G) = min $\{d(v) / v \in V(G)\}$ and $\Delta(G) = \max \{d(v) / v \in V(G)\}$.

If all points of G have the same degree r then G is called a regular graph of degree r.

Hence, irregular graph (G) = Δ (G).

A regular graph of degree 3 is called a cubic graph.

Subgraphs: A graph H = (V_1, X_1) is called a subgraph of G (V, X) if $V_1 \subseteq V$ and $X_1 \subseteq X$. H is a subgraph of G then we say that G is a super graph of H.

H is called a spanning graph of G if $V_1 = V$.

H is called an induced subgraph of G if H is the maximal subgraph of G with point set V_1.

i.e. if H is an induced subgraph of G then two points are adjacent in H if and only if they are adjacent in G.

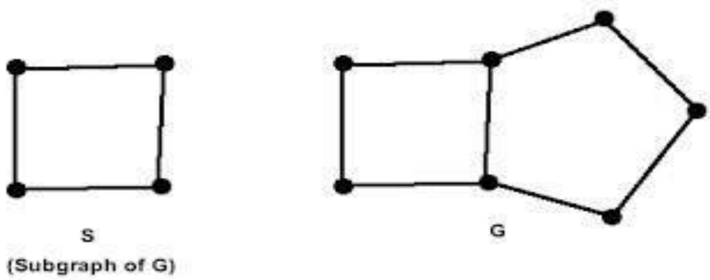

FIGURE 4. Subgraphs

Removal of a point: Let G = (V, X) be a graph and $v \in V$.

The subgraph of G obtained by removing the point v and all the lines incident with v is called the subgraph obtained by the removal of the point v and is denoted by G – v.

i.e. If G – v = (V_1 , X_1) then V_1 = V – { v} and X_1 = { x / x ∈X and x is not incident with v }. i.e. G – v is an induced subgraph of G.

Removal of a line: Let G = (V, X) be a graph and x ∈ X. Then G – x = (V, X –{x}) is called the subgraph of G obtained by the removal of the line x .

i.e. G – x is a spanning subgraph of G which contains all the lines of G except the line x.

Isomorphism: Two groups G_1 = (V_1, X_1) andG_2= (V_2, X_2) are said to be isomorphic if there exists a bijection f :$V_1 \rightarrow V_2$ such that u, v ∈V_1 are adjacent in G_1 if and only if f(u) , f(v) ∈V_2 are adjacent in G_2.

If $G_1$is isomorphic to G_2 then we write $G_1 \cong G_2$. The map f is called an isomorphism from G_1toG_2.

Automorphism*:* An isomorphism of a group onto itself is called an automorphism of G.

Remark*:* The set of all automorphism of G is a group. This group is denoted by Γ(G) and is called the automorphism group of G.

Vertex Covering: A covering of a graph G = (V, X) is a subset K of V such that every line of G is incident with the vertex in K. A covering K is called a minimum covering if G has no covering K_1 with $|K_1| < |K|$. The number of vertices in a minimum covering of G is called the coveringnumber of g and is denoted by β. A line covering of a graph G = (V, X) is a subset L of X such that every vertex is incident with a line of L. The number of lines in a minimum line covering of G is called the line covering number of G and is denoted by β_1. A set of lines is called independent if no two of them are adjacent. The number of lines in a maximum independent set of lines is called the edge independent number and is denoted by α_1. ***Walk:*** A walk of a graph G is defined as a finite alternating sequence of points and lines of the formv_0, x_1, v_1, x_2, v_2, x_3, v_3, ... , v_{n-1}, x_n, v_n beginning and ending with points such that each line $x_n$is incident with v_{n-1} and v_n.

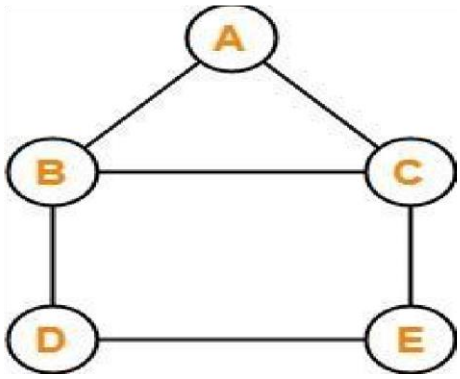

FIGURE 5. Walk

Closed Walk and Open Walk: A walk which begins and ends at the same point is called a closed walk.i.e. a v_0 – v_n walk is called walk if v_0= v_n.A walk that is not closed is called an open walk.

Cycle: A closed walk in which no point except the terminal point appearmore than once is called a cycle.

Aclosedwalkv_0, x_1, v_1, x_2, v_2, x_3, v_3, ... , $v_n = v_0$ in which n ≥ 3 and v_0, x_1, v_1, x_2, v_2, x_3, v_3, ... , $v_{n-1}$are distinct is called a cycle of length n. The graph consisting of cycle of length n is denoted by C n. C 3 is called a triangle.

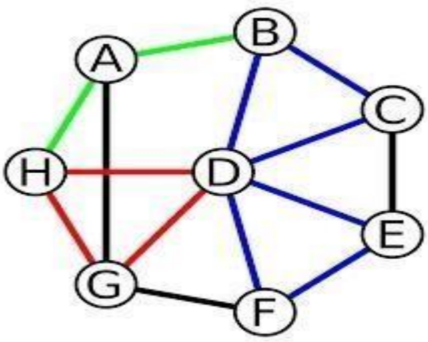

FIGURE 6. Cycle

Trial: A walk is called a trial if all its lines are distinct.
Path: A walk is called a path if all its points are distinct.

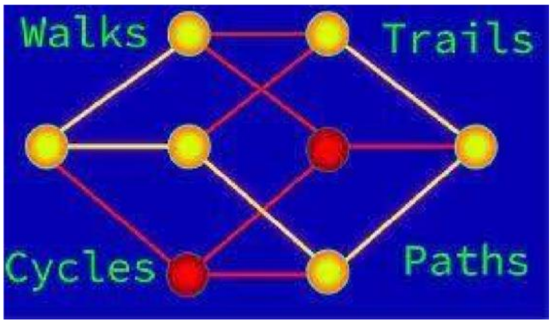

FIGURE 7. Path

Note: 1).Every path is a trial and a trail need not be a path. 2). The graph consisting of a trial with n points is denoted by p_n . 3).The length of a path in which the number of lines in the path.

2. REPRESENTATION OF JOB ORIENTED WEB SITES BY EDGE COLORING OF FUZZY GRAPHS

Fuzzy graph coloring problem has various practical applications. One of them is in job oriented web sites. Job oriented web sites are useful in recent years. A lot of recruiters and job seekers gather in these web sites. Fuzzy graphs represent each web site. The following example shows a step by step method of representation of web sites and edge coloring of fuzzy graphs. By looking at the color of a fuzzy graph, an applicant could understand how many companies suit his/her expertise. Furthermore, on the other side, a company finds a suitable applicant. Thus, the representation of job oriented web sites can be prepared in an easier way using the edge coloring of fuzzy graphs. Let us assume such a small web site, where *N* number of candidates are registered for jobs with their BioData, and *M* number of companies (recruiters) have registered a certain number of vacancies and the brand value of the company for appropriate candidates (see Fig. 1).

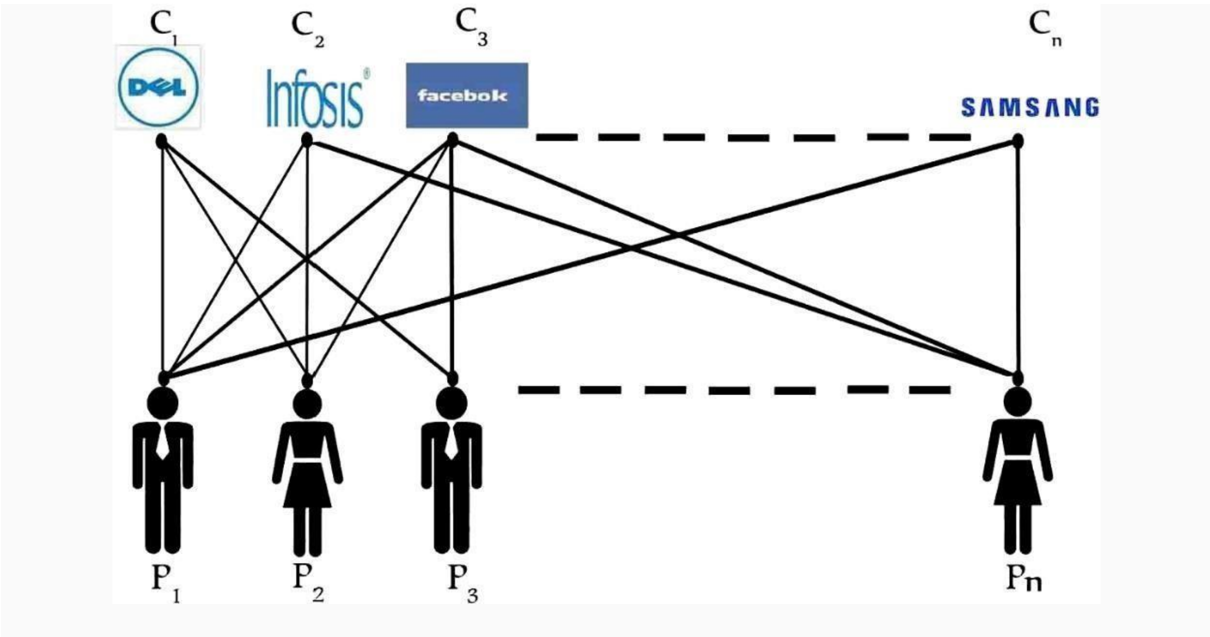

FIGURE 8. Representation of companies and applicants in a job-oriented web site

If an applicant's eligibility suits a company's demand, it is obvious that there exists a relation between them. Here, the companies and candidates are represented as vertices and their links as edges. Now, the membership values of corresponding vertices of the company may depend on the following issues: Salary; Company brand value; Product value; Job security; Medical benefits; Car benefits; Insurance benefits; Accommodation benefits; Service

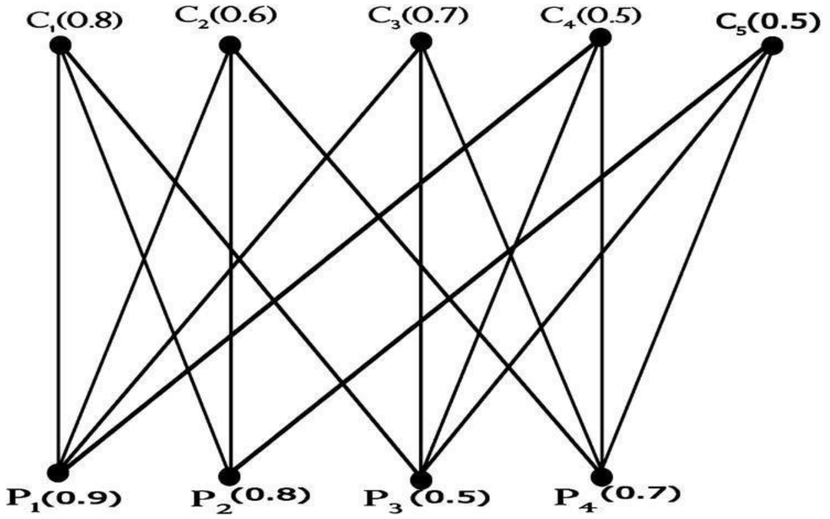

FIGURE 9. Fuzzy graph of the job-oriented web site

rule; Service hours; Job responsibility. And the membership values of vertices to the corresponding applicant depend on following parameters. Academic qualification; Experience; Language; Communication skill; Age; Salary requirement; Compensation; Behavior.

The membership values of edges depend on matching the companies' profiles with the applicants' profiles. Then, the relationship between companies and applicants is shown as a fuzzy bipartite graph, shown in Fig. 2. The company may shortly find suitable candidates by using the concept of strong chromatic index. Suppose, in particular, 5 companies and 4 applicants are registered on this job oriented web site (see Fig. 2). Thus, a fuzzy bipartite graph is considered where the vertices are the companies C1,C2,C3,C4,C5 and the applicants, P1,P2,P3,P4. There is an edge if there exists a particular matching. So the edges are (C1,P1), (C1,P2),(C1,P3), (C2,P1), (C2,P2), (C2,P4), (C3,P1), (C3,P3),(C3,P4), (C4,P1), (C4,P3),(C4,P4), (C5,P1), (C5,P2), (C5,P3), (C5,P4).The membership value of vertices C1,C2,C3,C4,C5 depend on the salary, company's brand value, product value etc., as stated above. The membership values of the vertices P1,P2,P3,P4 depend on qualification, experience, etc., as stated above. The membership values of edges depend on matching the companies with the applicants. For example, in a particular case, the membership values of vertices C1,C2,C3,C4,C5 are taken as 0.8,0.6,0.7,0.5,0.5, respectively. The membership values of vertices P1,P2,P3,P4 are considered as 0.9,0.8,0.5,0.7, respectively. Also the membership value of the edges (C1,P1), (C1,P2), (C1,P3), (C2,P1), (C2,P2), (C2,P4), (C3,P1), (C3,P3), (C3,P4), (C4,P1), (C4,P3), (C4,P 4), (C5,P1), (C5,P2), (C5,P3), (C5,P4) are taken as 0.7, 0.6, 0.5, 0.6, 0.5, 0.4, 0.7, 0.4, 0.6, 0.4, 0.4, 0.5, 0.4, 0.3, 0.2, 0.3, 0.2, respectively. When a company or an applicant logins into this web site, then they get a fuzzy subgraph like Fig. 2. Particularly, if an applicant P2 logins into this web site, then he/she gets a fuzzy subgraph shown in Fig. 3(a). Also, if a company, in particular, C3 logins into this web site, it gets another fuzzy subgraph shown in Fig.3(b).

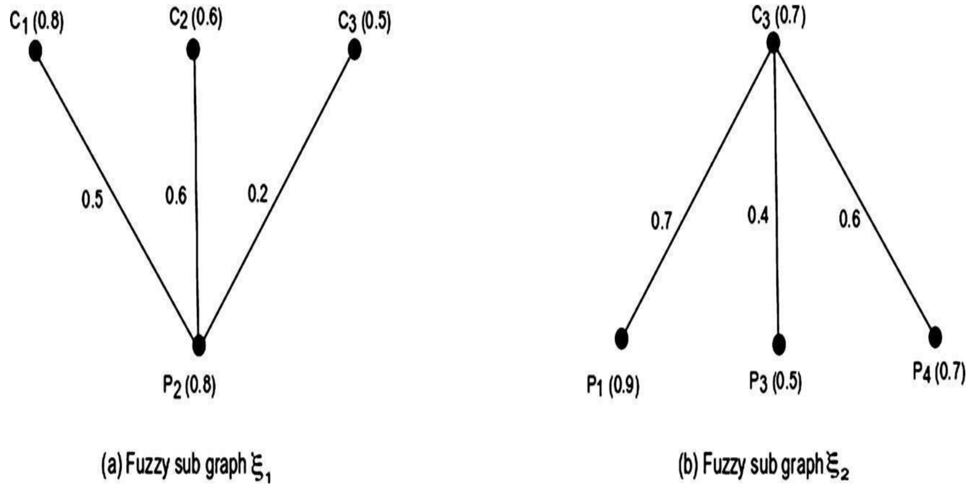

FIGURE 10. Fuzzy sub graphs of the fuzzy graph of Fig. 2.

Scheduling: Vertex coloring models to a number of scheduling problems. In the cleanest form, a given set of jobs need to be assigned to time slots, each job requires one such slot. Jobs can be scheduled in any order, but pairs of jobs may be in conflict in the sense that they may not be assigned to the same time slot, for example because they both rely on a shared resource. The corresponding graph contains a vertex for every job and an edge for every conflicting pair of jobs. The chromatic number of the graph is exactly the minimum make span, the optimal time to finish all jobs without conflicts. Details of the scheduling problem define the structure of the graph. For example, when assigning aircraft to flights, the resulting conflict graph is an interval graph, so the coloring problem can be solved efficiently. In bandwidth allocation to radio stations, the resulting conflict graph is a unit disk graph, so the coloring problem is 3-approximable.

Job Scheduling: Here the jobs are assumed as the vertices of the graph and there is an edge between two jobs 1 they cannot be executed simultaneously. There is a 1-1 correspondence between the feasible scheduling of the jobs and the colorings of the graph.

Representation of Traffic Light Problem:

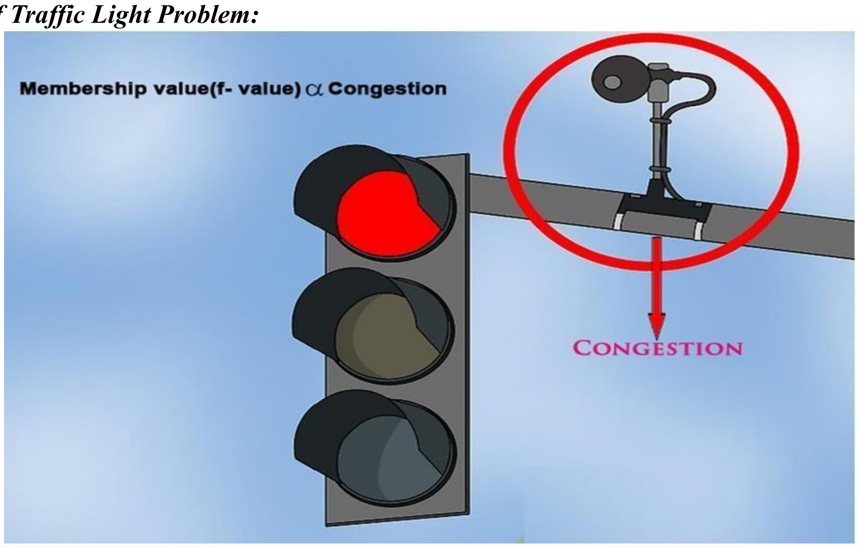

FIGURE 11. CCTV for collection of data

The control policy of the traffic light depends mainly on the number of vehicles in the intersection line. If the traffic flow in the intersection line is high then there is a possibility of accident. When the number of vehicles in the intersection line is low then there may be less possibility of accident. The concept of accident and number of vehicles in each line could be fuzzy and it could be graduated. This graduation, which does not need to be numerical, is associated to the desired security level for the traffic. Here we represent each traffic flow with a fuzzy edge whose membership value depends on the number of the vehicles in that path. Two fuzzy vertices are adjacent if the corresponding traffic flows cross each other then there is a possibility of accident. Possibility of accident value will depend on vertex membership value. The maximum-security level is attained when all lanes are considered to be in intersection with each other and the number of vehicles in each line is also high. So, graph will be a complete graph. In this case, the chromatic number is the number of lanes and the control policy of the lights assure that only one movement is allowed in any slot of the cycle. On the other hand, the minimum-security level is attained when the intersection edge set is empty; in this case, the chromatic number is 1 and all movements are allowed at any instant. Since the four right turns do not interfere with the other traffic flows, they can safely be dropped from our discussion. Membership values can be represented by symbolic name H for high, M for medium for low respectively. One of the most useful real-life applications of the edge coloring is in traffic light problems. Traffic light system uses three standard colors: red, amber (yellow), green, following the universal color code. The green light allows traffic to proceed in the denoted direction. The amber (yellow) light warns that more precautions should be taken to cross the road. The red signal prohibits any traffic from proceeding. However, in the present traffic light system, a traveler does not know how much the traffic is congested. This limitation has to be removed here. Traffic light system will be modified by the edge coloring of fuzzy graphs. It takes every route as a fuzzy vertex. An edge between two vertices is drawn if the routes have a junction. The edge membership values are calculated based on the congestion of routes and the road condition. The data is collected from real time CCTV (see Fig. 4).

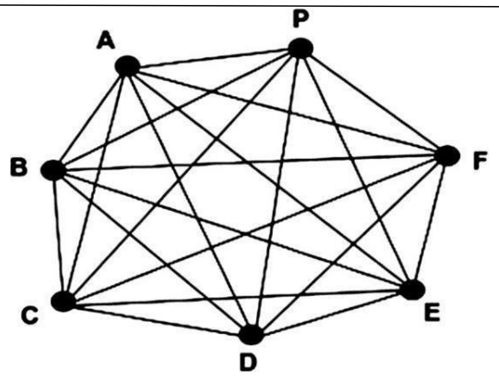

FIGURE 12. Fuzzy graph of seven-point crossing.

This picture is converted into a fuzzy graph (see Fig. 5) . Now, the membership value of the edges in the figure PA, PB, PC, PD, PE, PF, AB, AF, AE, AD, AC, BC, BD, BE, BF, CD, CE, CF, DE, DF, EF are calculated from real time CCTV showing the road conjunction and the road condition. A traveler from P road can visit any route of this crossing if all routes are open at that time. So a fuzzy subgraph can be constructed against the node P to

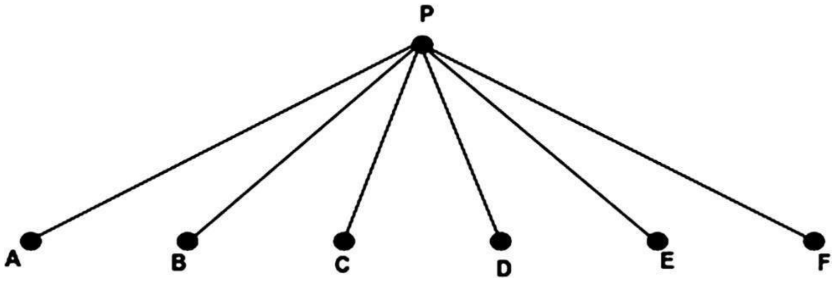

FIGURE 13. Fuzzy subgraph.

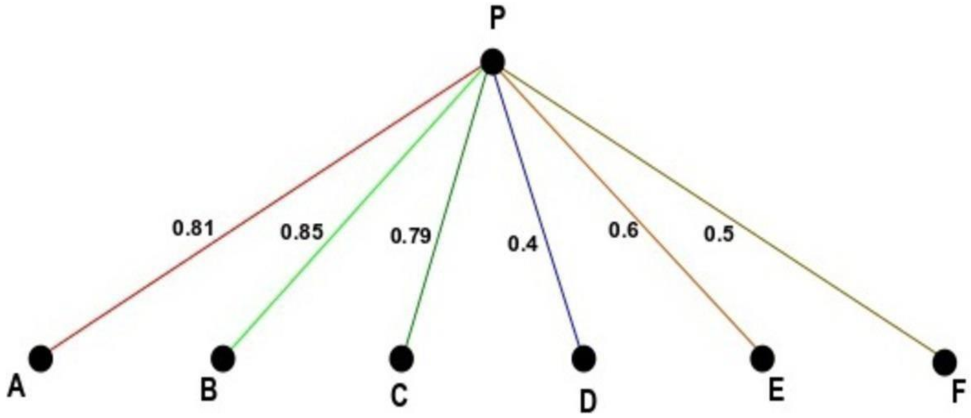

FIGURE 14. Edge coloring of fuzzy graph.

represent the travelers possible visit, whose edges are PA, PB, PC, PD, PE, PF (see Fig. 6).

Let us consider at a particular time the membership value of the edges PA, PB, PC, PD, PE, PF be 0.81,0.85,0.79,0.4,0.6,0.5, respectively. Next, this fuzzy subgraph is colored using the edge coloring technique (see Fig. 7). In this subgraph, the chromatic index is 6 and the weight is (0.81+0.85+0.79+0.4+0.6+0.5)=3.95. When a traveler is travelling the P road, it would be helpful, if all the information about the next crossing was displayed in two or three display boards before the crossing. From the chromatic index the traveler can understand how many roads are open at the time of crossing, and from the weight, the total traffic condition. For this purpose, the congestion of the next crossing is to be represented in terms of percentage. Here, f-value, i.e. 0.81 indicates that the congestion is 81%. Displaying percentage is helpful in order to understand the congestion of a target road. So with the help of the coloring of the fuzzy subgraph, the traveler would get an idea before crossing about the present condition of the target road. With the help of the chromatic index, the traveler also understands the situation of the target road, i.e. whether the road will open or close.

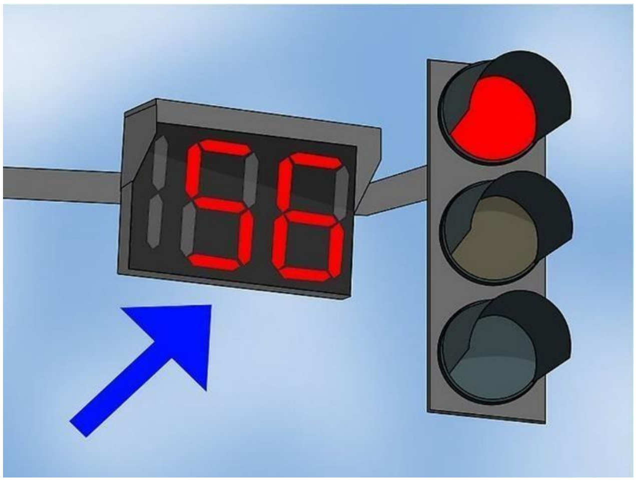

FIGURE 15. Red signal with percentage of mixing of colors.

Also we can use this depth of color in red ($f(c_i)$) signal with percentage (see Fig. 8). If the membership value is increased, then the density of the red signal will be increased. On the other hand, the green signal can be used with the membership value $1-f_1(c_i)$ which is the complement of the depth of red color. The denser the green color is, the less congested the route is. So the traveler can understand how much danger there is or how much time will be spent to cross. The stopping time can be fixed with the help of the membership value of the red signal. If the membership value of the red color is increased, then the stopping time will decrease.

3. CONCLUSION

In this study, a concept of edge coloring has been introduced. A related term, chromatic index, is also defined in a different way. A weight is associated with each of the chromatic indexes. This weight might be defined in different ways but the proposed method mentions the depth of the colors to be used to color a graph. At the end of this paper, the traffic light problem is updated. In that problem, if the subgraph is uploaded for online traffic condition, then the input graph will be compatible with Google Traffic indicating system. It is seen that Google Traffic system calculates the data from the flow of mobiles. But here, membership values are calculated from realtime CCTV. Google Traffic uses three or four colors to represent the condition of the traffic, but the proposed method uses the color density with a percentage, which is much more helpful to the traveler. Thus, a user can easily understand the present condition of the traffic. There are many real field problems which can be solved using this technique of coloring, such as transportation problems, social networking problems, sport modeling. The proposed method may be implemented empirically in the existing traffic light systems. The theoretical approach of edge coloring will be beneficial to other graph coloring problems as well. In future studies, more uncertainty may be represented through generalized fuzzy graphs.

REFERENCES

[1]. Bershtein, L.S., Bozhenuk, A.V. (2001). Fuzzy coloring for fuzzy graphs. In: lEEE International Fuzzy Systems Conference, pp. 1101–1103.

[2]. Kishore, A., Sunitha, M.S. (2014). Strong chromatic number of fuzzy graphs. Annals of Pure and Applied Mathematics, 7, 52–60.

[3]. Kishore, A., Sunitha, M.S. (2016). On Injective Coloring of Graphs and Chromaticity of Fuzzy Graphs. LAP Lambert Academic Publishing.

[4]. Akram. M, Karunambigai. M. G. and Kalaivani. O. K., Some Metric Aspects of Intuitionistic Fuzzy Graphs, World Applied Sciences Journal, 17 (2012), 1789-1801.

[5]. Akram. M. and Bijan Davvaz Strong Intuitionistic fuzzy graphs, Faculty of Sciences and Mathematics, 26 (2012) 177196.M.S. Sunitha and S. Mathew, Fuzzy graph theory: a survey, Annals of Pure and Applied Mathematics 4(1)(2013)92-110.

[6]. A.K.Bincy and B.Jeba Presitha , Graph Coloring and its Real Time Applications an Overview, International Journal of Mathematics And its Applications, Volume 5, Issue 4–F (2017), 845–849. ISSN: 2347-1557