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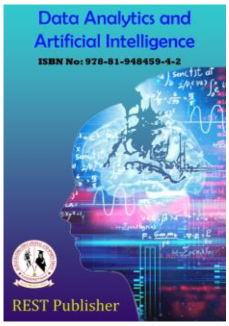

A Study on Mixed Inverse Center-Smooth Set of Some Graphs and its application

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Abstract. *For S is a dominating set of G and V-S V(G) of a center smooth graph Gis called amixed inverse center smooth set if (i) For every vεV-S, |N[v]∩V(G)| 1(mod p) and (ii) Every elementuεS is either adjacent or incident to an element of V-S. The number of vertices in a mixed inversecenter smooth set is called the mixed inverse center smooth number and it is denoted by γmcs(G).Inthis paper, we introduce the new concept of mixed inverse center smooth number and establish someresults on this new parameter. Also, we determine the bounds of γmcs- set of some graph classes.*

Keywords: *Center smooth graph, Inverse dominating set, Center-smooth set, mixed inverse centerSmooth set*

1. INTRODUCTION

We consider only finite simple undirected connected graphs. For the graph G, $V(G)$ denotes itsvertex set and $E(G)$ denotes its edge set. As usual, p=|V| and q= |E| denote the number of vertices andedges of a graph G, respectively. For a connected graph and a pair u, v of vertices of G, the *distance* $d(u,v)$ between u and v is the length of a shortest u-v path in G.The *degree* of a vertex u, denoted by *deg(u)*is the number of vertices adjacent to u. A vertex u of a graph G is called a *universal vertex* if u isadjacent to all other vertices of G. A graph G is *universal graph* if every vertex in G is universal vertex.For example, the complete graph $K_p$is universal graph. The set of all vertices adjacent to u in a graph G,denotedby$N(u)$,isthe*neighborhood*ofthevertex$u$.The*eccentricity*$e(u)$ofavertex$u$isthedistanceto a vertex farthest from u. Thus, $e(u) = max\{ d(u, v)/v\ V(G)\}$. A vertex v is an *eccentric vertex* of u if$e(u)= d(u, v)$. The radius $r(G)$ is the minimum eccentricity of the vertices, whereas the diameter*diam*(G) is the maximum eccentricity. The center of G, $C(G)=\{ vV(G)/ e(v)= r(G)\}$.

Definition 1.1

A *clique* of a graph G is a complete subgraph of G, and the clique of largest possible size isreferred to as a maximum clique (which has size known as the clique number $\omega(G)$).

Definition 1.2

A *vertex cover* of G is a set of vertices that covers all the edges. The vertex covering number (G) isminimum cardinality of a vertex cover.

Definition 1.3

The *vertex cut* or separating set of G is a set of vertices whose removal results in a disconnected.The *connectivity* or *vertex connectivity* of a graph G, denoted by $\kappa(G)$(where G is not complete) is thesize of a smallest vertex cut.

Definition 1.4

The *S-eccentricity* $e_S(v)$ of a vertex v in G is . The *S-center* of G is $C_S(G) = \{vVxV\}$.

Example 1

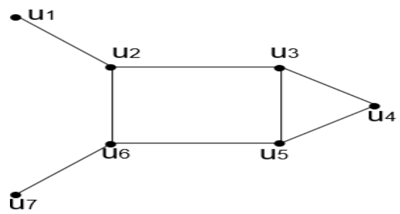

Fig.1. Center-smooth graph

Infigure1,S={u_1,u_3,u_6}and$V$-S={u_2,u_4,u_5,u_7}.The$S$-eccentricitye_S(u_1)=3,e_S(u_2)=1,e_S(u_3)=3,e_S(u_4)=3, e_S(u_5)=3, e_S(u_6)=2, e_S(u_7)=3. Then the S-center C_S (G) = {u_2}.

Definition 1.5
The S_1-*eccentricity*,of a vertex v in S is . The S_1 *center* of G is(G) ={vεV for all xεV}.

Example 2
Infigure1,S={u_1,u_3,u_6}and$V$-S={u_2,u_4,u_5,u_7}.TheS_1-eccentricity(u_1)=3,(u_2)=1,(u_3)=3,(u_4)=3,(u_5)=3,(u_6)=2,(u_7)=3. Then the S_1-center, (G) = {u_2}.

Definition 1.6
Let G be a graph and S be a proper set of G. G is called a *center-smooth graph* if C_S (G) (G) andthe set S is said to be a *center-smooth set*.

Example 3: In figure 1, C_S (G) = {u_2} = (G).

Definition 1.7
An SV is a *dominating set* in G if every vertex not in S is adjacent to at least one vertex of S. If S isa dominating set then V- S (*inverse dominating set*) need not be a dominating set.

Definition 1.8
A set S is *independent* if no two vertices in it are adjacent. An *independent dominating set* of G is aset that is both dominating and independent in G. *Independence domination number* ($\gamma_i(G)$) (G) of G isthe maximum (minimum) cardinality taken over all independent dominating sets of G.

Definition 1.9
A set S is called 1-*dominating set* if for every vertex in V-S, there exists exactly one neighbor in S.The minimum cardinality of a 1-dominating set is denoted by $\gamma_1(G)$.

Definition 1.10
Let Sbe a dominating set of center smooth graph G. Then the *Restrict*-S^c(RS^c) set of a graph G isdefinedbyRS^c andthenumberofRS^c-setof$G$isdenotedbynR(G).IfRS^c-setisindependentsetthen the number of RS^c- set of G is denoted by niR(G).

Definition 1.11
Let Sbe a dominating set of G and $RS^c \subseteq V(G)$. Then the set $RS^c$is called a *center smooth 1^{c}dominating set* of a center smooth graph G if for every vertex in S^chas at least one neighbor in S. Thenumber of vertices in RS^c of a center smooth graph G is called *center smooth 1^{c}domination number* andit is denoted by cs(G).

2. RESULT ON MIXED INVERSE CENTER SMOOTH SET

Definition 2.1

Let$S$beadominatingset.Then$V$-$S \subseteq V(G)$ofacentersmoothgraph$G$iscalleda*mixedinversecenter smooth set* if

(i) For every $v \in V$-S, $|N[v] \cap V(G)|$1(mod p) and (ii)Every element $u \in S$is either adjacent or incident to an element of *V-S*.

The number of vertices in a mixed inverse center smooth set is called the mixed inverse center smooth number and it is denoted by $\gamma_{mcs}(G)$.

Observations 2.1.

For any connected graph G,$cs(G) \leq \gamma_{mcs}(G)$.

Example:

Consider the following figure *G*

(1) There are graphs with $\gamma_{mcs}(G) > cs(G)$.

In figure 1,we take$S = \{u_1,u_3,u_6\}$ and $V\text{-}S = \{u_2,u_4,u_5,u_7\}$.

$\Rightarrow \gamma_{mcs}(G)=4andcs(G)=2$.

(2) There are graphs with $\gamma_{mcs}(G) = cs(G)$.

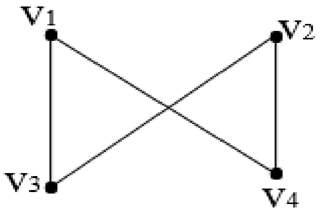

Fig.2. $K_{2,2}$ graph

Take, $V\text{-}S = \{v_2, v_3\}$, $S= \{v_1, v_4\}$.

$\Rightarrow cs(G)=\gamma_{mcs}(G)=2$.

Theorem 2.1.

For any connected graph G with $\gamma(G) =(G)$,$\gamma_{mcs}(G)+(G) =p$ iff$G= K_{1, p-1}$.

Proof:Suppose$G=K_{1,p-1}$.Let$u \in V(G)$dominatesalltheverticesof*G*. Since,$\gamma(G)=(G)=1$.Let$V\text{-}S=$ $\{v, w, x, y\}$ be a mixed inverse center smooth set of *G*. Therefore *G* has $|V\text{-}S|+1$ vertices. Which implies$p = \gamma_{mcs}(G) +(G)$. Conversely, suppose that $\gamma_{mcs}(G) + (G) = p$. Let $S = \{v\}$ be a dominating set of *G*. Since$\gamma(G) =(G)$, then a vertex *v* covers all the edges of *G*. Therefore $(G) = \{v\}$ and so $\gamma(G) =(G)=\{v\}$.Therefore a vertex *v* of degree p-1. Hence it follows that $G= K_{1, p-1}$. ■

Theorem 2.2.

For any graph *G*, $\gamma_{mcs}(G) = p$-1 iff*G* has a universal vertex.

Proof: Suppose $\gamma_{mcs}(G) = p$-1. Now we prove that *G* has a universal vertex. Let *V-S* be the γ_{mcs}-set of *G*.On the contrary, assume that *G* has not a universal vertex. Then there exists two non-adjacent vertices $u,v \in S$such that *x* is adjacent to both *u* and *v* and *y* is adjacent to *v* where *x* and *y* are adjacent. Clearly, *V-S*is a mixed inverse center-smooth set of *G* and $\gamma_{mcs}(G) \leq p$-2, which is a contradiction. Hence, *G* has auniversal vertex. Conversely, suppose that *G* has a universal vertex. Hence an universal vertex $u \in V(G)$dominate all other vertices in *G*. Therefore, $\gamma_{mcs}(G) =p$-1. ■

Theorem 2.3.

Let *V-S* be a mixed inverse center-smooth set of *G*. If every vertex *v* in *S* which is adjacent to allthe vertices of *V-S*. Then $\gamma_{mcs}(G) + \gamma(G)=p$.

Proof:Let*V-S*beamixedinversecenter-smoothsetof*G*.Let$v \in S$.Then*v*dominatesalltheverticesin *V-S*.Therefore$\gamma(G)=|S|=1$.

Then, $\gamma_{mcs}(G) = |V\text{-} S|$

$$\Rightarrow \gamma_{mcs}(G)+|S|=p$$

$$\Rightarrow \gamma_{mcs}(G)+\gamma(G)=p.$$

Corollary 1: If a graph G has p-1 pendent vertices, then $\gamma_{mcs}(G) + \gamma(G)=p$.

Proof:InagraphG,eachpendentvertexisadjacenttoavertexvinGofdegreeisp-1.Thatis,avertexv dominates all the other vertices in G. Therefore $\gamma(G) =|S| =1$. Hence, it follows that V-S is the mixedinverse center smooth set of G. $\gamma_{mcs}(G) =|V-S| =p- \gamma(G)$.Hence, $\gamma_{mcs}(G) +\gamma(G) =p$.

Corollary2: IfV-Sisamixedinversecenter-smoothsetofGandSisindependent,then$\gamma_{mcs}(G)+\gamma(G)$

$= p$.

Proof:

Let V-S be a mixed inverse center-smooth set of G. Further, let S is dominating set of G which isindependent. Since, $|S|=|V|- |V- S|$. Clearly,$\gamma(G)=p-\gamma_{mcs}(G)$. Hence the result. ■

Theorem 2.4.
If G has a universal vertex which is not vertex cover, then $\gamma_{mcs}(G) +cs(G)=2(p-1)$
Proof:Let G be a universal vertex which is not vertex cover, then v dominates all other vertices in G.Clearly, $\gamma_{mcs}(G) =p-1$ and $cs(G)= p-1$ .Therefore $\gamma_{mcs}(G)+cs(G)=2(p-1)$. ■

Theorem 2.5.
$\gamma_{mcs}(K_{m, n}) =p-2$.

Proof: For $V(K_{m, n}) =V_1 \cup V_2$, $|V_1| =q$ and $|V_2|= p$ such that each element of V_1 is adjacent to every vertex of$V_2$and vice versa. Let $V\text{-}S = \{u, v\}$, $u\in V_1,v\in V_2$. Then, clearly u dominates all the vertices of V_2.Similarly,$v$dominatesalltheverticesofV_1.Hence$V$-Sisamixedinversecentersmoothsetand$K_{m,n}$has
$|V-S|+2$ vertices. That is, $p =|V-S|+2$. This implies $|V-S| =p-2$. Hence it follows that $\gamma_{mcs}(K_{m, n}) = p-2$.
Theorem 2.6.
If G is a triangle free graph of radius 2, then $cs(G) \leq \gamma_{mcs}(G)$.

Proof: Let $RS^{c}\subseteq V(G)$ be the center smooth 1^{c}dominating set of G . Further, let $v\in V(G)$ be adjacent tomore than one vertex of S. Then, by the definition of RS^{c}, v RS^{c}. Clearly, v dominates $N(v)$ and also thevertices of $N(v)$ are disconnected, Since G has no triangles. Further, v is adjacent to atleast one vertex in$V$-S. Therefore, $cs(G) \leq \gamma_{mcs}(G)$. ■

3. BOUNDS ON $\Gamma_{MCS}(G)$

Theorem 3.1

For any graph G with p vertices, $\gamma_{mcs}(G) \leq p- \mathcal{K}(G)$ where $\mathcal{K}(G)$ is the vertex connectivity of G.
Proof: Let$m$bethesetof$G$with$\mathcal{K}(G)$verticessuchthat$|m|=\mathcal{K}(G)$.Furtherlet$V$-Sbeamixedinversecenter smooth set of G and therefore $\gamma_{mcs}(G)+ |m|\leq p$ and hence, $\gamma_{mcs}(G)\leq p-\mathcal{K}(G)$. ■

Theorem 3.2
For a mixed inverse center smooth set of G, $\gamma_{mcs}(G) \geq p-\omega(G)$ where $\omega(G)$ is the clique number of G.**Proof:**Let$m$beasetofverticesin$G$suchthat$\langle m\rangle$iscompletewith$|m|=\omega(G)$.Furtherlet$V$-Sbeamixed inverse center smooth set of G.

$$\text{That is, } |V-S|+|m| \geq p$$

$$\Rightarrow \gamma_{mcs}(G)\geq p-|m|=p-\omega(G). ■$$

4. APPLICATIONS

Consider the problem of a number of communities (interlinked by a road network),whichhas to be served by a single hospital, police station or fire station. Let us now assume that the arc“lengths” $c_{ij}$of the graph G whose vertices represent the communities and whose arcs represent the road.Form the matrix corresponding to the travel times between these communities. This matrix is notnecessarily symmetrical. That is, $c_{ij} \neq c_{ji}$since the traveling times may be affected by slopes in the road, one-way streets, etc. In the case of locating a police station or a fire station, what is of interest is the time that is required to reach the most distant of these communities and the problem is, therefore, to locate the police (or fire station) so as to minimize this time. In the case of locating a hospital, what may be of interest is the time that an ambulance takes to reach the most distant community and return back to the hospital. Itisrequiredforsomereasonthatthesefacilitiesmustbelocatedinoneofthesecommunitiesandnot just any arbitrary point along the road. This location can be found by the absolute center. The concept can be used in tree flow networks. The centerv vertices can be treated as sources of the network.

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