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Shortcomings Of Existing Ranking Methods For Interval-Valued Intuitionistic Fuzzy Numbers

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Abstract
Ranking of alternatives plays an important role in the decision-making (DM) problems. There are lots of methods for ranking of interval-valued intuitionistic fuzzy numbers (IVIFNs) in literature. Therefore, the main objective of this study is to review the existing score and accuracy functions of IVIFNs and to find the shortcomings where these functions are unable to rank the numbers. This study gives a platform to develop a new ranking method for IVIFNs in future.
Keywords: Decision-making, Score function, Accuracy function, Shortcomings.

I. Introduction

In all fields of human life such as business, society, medical science, military, project evaluation etc., one of the most important problem is decision making (DM) problem. DM is the process to find an optimal alternative among the various feasible alternatives. There are several types of DM problems such as simple DM problem, multi-criteria decision making (MCDM) problem, multi objective decision making problem, multi-attribute decision making (MADM) problem etc. In MADM, a decision maker selects the best alternative(s) or rank the alternatives after qualitative and quantitative valuation on a finite set of mutual dependent or independent attributes. Traditionally, it has been assumed that all the information which accesses the alternative in terms of attribute and their corresponding weights are expressed in the form of crisp numbers. But day to day, it has been widely recognized that the most of problems are complex in nature and involve many conflicting nature of attributes, uncertainty and imprecise. Therefore, it is necessary to deal with the uncertainties and imprecise data so that the decision makers can give their decision accurately within a reasonable time. In this order, fuzzy set theory [17] and its extensions such as intuitionistic fuzzy set(IFS) [2], interval-valued IFS(IVIFS) [1] etc., have been widely used and successfully applied in different disciplines. For solving the DM issues, ranking of the alternatives play an important role. For this, Chen and Tan [5] introduced the concept of score and accuracy function for IFS. Afterwards, Xu [15] extended the idea of score and accuracy functions for IVIFNs. Nayagam et.al [8] defined the novel accuracy function. Sahin [12], Ye [16] proposed novel accuracy score and improved accuracy score function for IVIFNs respectively. Bai [3] proposed improved score function to rank IVIFNs. Garg [6] presented a generalized improved score function of IVIFS. Zhang and Xu [18] proposed the contrast degree and a score function for compare to IVIFNs. Nayagam et.al [9] developed a non-hesitance score function for ranking of IVIFNs that overcomes the drawbacks of Nayagam et al [8], Sahin [12], Ye [16], Zhang and Xu [18]. Joshi and Kumar [7], Priyadharsini and Balasubramaniam [11] defined a score and accuracy function for IVIFNs respectively. Wang and Chen [13] defined a score function and a DM approach for handling the MADM issues that overcomes the flaws of DM approach given by [4]. While, Wang and Chen [14] found some drawbacks of score function defined by Wang and Chen [13], and proposed a new score function with the application in MADM problems. A part from these, Zhang et al [19] defined the parameterized score function for ranking of IVIFNs. Recently, [10] defined the p-norm knowledge score function to rank the IVIFNs. In certain cases, however, the existing score and accuracy function do not provide adequate details on ranking of alternatives. Therefore, in this article, we have presented some counter examples to show the shortcomings of the existing score and accuracy functions of the IVIFNs. These counter examples will help to develop a new ranking method for IVIFNs. To accomplish the above targets, this paper is sorted out as: In Section 2, briefly introduces the concepts of IVIFS. Section 3 shows the existing score and accuracy functions of IVIFS theory. Section 4 presents the flaws of existing score and accuracy functions. Finally, Section 5 concludes the paper.

2. Preliminaries

In this segment, a few fundamental concepts related to the IVIFS theory with existing score and accuracy function are characterized as follows:

Definition 1: [2] An IVIFS A is defined as

$$\mathrm{A} = \{\langle x, [\tau_{\mathrm{A}}(x), \eta_{\mathrm{A}}(x)], [\theta_{\mathrm{A}}(x), \upsilon_{\mathrm{A}}(x)] \rangle \mid x \in X\}$$

where, $\tau_A(x), \eta_A(x), \theta_A(x), \upsilon_A(x) \in [0,1]$, represents membership and non-membership degrees of x to A respectively, such that $0 \le \eta_A(x) + \upsilon_A(x) \le 1$ holds for $\forall x$. Usually the pair $\langle[\tau,\eta],[\theta,\upsilon]\rangle$ is called an IVIFN.

Definition 2: Let $A = \langle[\tau,\eta],[\theta,\upsilon]\rangle$, $A_1 = \langle[\tau_1,\eta_1],[\theta_1,\upsilon_1]\rangle$ and $A_2 = \langle[\tau_2,\eta_2],[\theta_2,\upsilon_2]\rangle$ be any three IVIFNs, then

a) $A_1 \subseteq A_2$ if $\tau_1 \le \tau_2, \eta_1 \le \eta_2, \theta_1 \ge \theta_2$ and $\upsilon_1 \ge \upsilon_2$.

b) $A_1 = A_2$ iff $A_1 \subseteq A_2$ and $A_1 \supseteq A_2$.

c) $A^c = \langle[\theta,\upsilon],[\tau,\eta]\rangle$.

Definition 3: [19]For an IVIFN $A = \langle[\tau,\eta],[\theta,\upsilon]\rangle$, normalized entropy measure is defined as

$$E(A) = (\eta - \tau) + (\upsilon - \theta) \tag{1}$$

Definition 4:[10]For an IVIFN $A\langle[\tau,\eta],[\theta,\upsilon]\rangle$, generalized p-norm knowledge measure of is defined as follows:

$$K_{ivF}(A) = \frac{1}{2^{1/p}+1}\left[\frac{\tau^p+\eta^p+\theta^p+\upsilon^p}{2} + \frac{(\tau+\theta)^p+(\eta+\upsilon)^p}{2} + \left|\frac{\tau^p-\theta^p+\eta^p-\upsilon^p}{2}\right|^{1/p}\right], (p=1,2,\ldots) \tag{2}$$

3. Existing Score and Accuracy Functions of IVIFNs

For comparing two numbers, various researchers defined the various ranking methods like as score and accuracy functions. Therefore, some existing score and accuracy function for an IVIFN $A = \langle[\tau,\eta],[\theta,\upsilon]\rangle$ are defined as:

i) Score function[15]:

$$S(A) = \frac{\tau+\eta-\theta-\upsilon}{2} \tag{3}$$

ii) Accuracy function[15]:

$$H(A) = \frac{\tau+\eta+\theta+\upsilon}{2} \tag{4}$$

iii) Novel accuracy score function[16]:

$$M(A) = \frac{\tau-(1-\tau-\theta)+\eta-(1-\eta-\upsilon)}{2} \tag{5}$$

iv) New novel accuracy score function[8]:

$$L(A) = \frac{\tau+\eta-\upsilon(1-\eta)-\theta(1-\tau)}{2} \tag{6}$$

v) Improved accuracy function [7]:

$$T(A) = \frac{\tau(1-\theta)+\eta(1-\upsilon)}{2} \tag{7}$$

vi) Improved accuracy score function[12]:

$$K(A) = \frac{\tau+\eta(1-\tau-\theta)+\eta+\tau(1-\eta-\upsilon)}{2} \tag{8}$$

vii) Improved score function[3]:

$$I(A) = \frac{\tau+\tau(1-\tau-\theta)+\eta+\eta(1-\eta-\upsilon)}{2} \tag{9}$$

viii) New accuracy function[11]:

$$P(A) = \frac{\tau+\eta+\theta-\upsilon}{4} \tag{10}$$

ix) Generalized score function[6]:

$$GIS(A) = \frac{\tau+\eta}{2} + k_1\tau(1-\tau-\theta) + k_2\eta(1-\eta-\upsilon), \tag{11}$$

where, $k_1, k_2 \ge 0$ and $k_1 + k_2 = 1$.

x) Non hesitance score function[9]:

$$J(A) = \frac{\tau+\eta+\theta-\upsilon+\tau\eta+\theta\upsilon}{3} \tag{12}$$

xi) New score function[13]:

$$S_{wc}(A) = \frac{\tau+\eta+\sqrt{\eta\upsilon}(1-\tau-\theta)+\sqrt{\tau\theta}(1-\eta-\upsilon)}{2} \tag{13}$$

xii) Contrast degree function[18]:

$$c(\mathrm{A}) = \frac{1}{2}\left(\frac{(\tau-\theta)+(\eta-\upsilon)}{2}+1\right) \quad (14)$$

xiii) Accuracy score function [18]:

$$f(\mathrm{A}) = \frac{1}{2}\left[\frac{(\tau-\theta)+(\eta-\upsilon)(1-\tau-\theta)}{2}+\frac{(\eta-\upsilon)+(\tau-\theta)(1-\eta-\upsilon)}{2}+1\right] \quad (15)$$

xiv) Novel score function [14]:

$$S_{NWC}(\mathrm{A}) = \frac{(\tau+\eta)(\tau+\theta)-(\theta+\upsilon)(\eta+\upsilon)}{2} \quad (16)$$

xv) Novel accuracy function[14]:

$$H_{NWC}(\mathrm{A}) = \frac{(1-\tau+\eta)(1-\tau-\theta)-(1-\theta+\upsilon)(1-\eta-\upsilon)}{2} \quad (17)$$

xvi) Parameterized score function[19]:

$$G(\mathrm{A}) = \left[\frac{\sin(c(\mathrm{A}))}{\sin 1}.\frac{\cos(H(\mathrm{A}))}{\sin 1}.\frac{\cos(E(\mathrm{A}))-\cos 1}{1-\cos 1}\right]^{\frac{1}{3}} \quad (18)$$

xvii) Score function based on p-norm knowledge[10]:

$$S_{ivF}(\mathrm{A}) = \begin{cases} \left[\dfrac{e^{C\left(\frac{\tau+\eta-\theta-\upsilon}{2}\right)}-1}{e^{C\left(\frac{\tau+\eta-\theta-\upsilon}{2}\right)}+1}\right]K_{ivF}(\mathrm{A}) & \text{for } [\tau,\eta]\neq[\theta,\upsilon] \\ \varepsilon K_{ivF}(\mathrm{A}) & \text{for } [\tau,\eta]=[\theta,\upsilon] \end{cases} \quad (19)$$

where, C ? 0 and $0 < \varepsilon =$ 1.

4. Shortcomings of the Existing Score and Accuracy Functions

Example 1: Few illustrations are given to show that the existing score and accuracy functions are unable to rank the alternative as follows: Consider two IVIFNs $\mathrm{A}_1 = \langle[0.0, 0.0],[0.2, 0.3]\rangle$ and $\mathrm{A}_2 = \langle[0.0, 0.0],[0.1, 0.4]\rangle$ and applying the existing score and accuracy functions defined in Eqs. (3), (4),(5), (6), (7), (8),(9),(11), (13), (14), and (19), then obtained results are summarized in Table 1.

Table 1

	$S(.)$	$H(.)$	$M(.)$	$L(.)$	$T(.)$	$K(.)$	$I(.)$	$GIS(.)$	$S_{wc}(.)$	$c(.)$	$S_{ivF}(.)$	$I(.)$
A_1	-0.25	0.25	-0.75	-0.25	0.00	0.00	0.00	0.00	0.00	0.375	-0.25	0.00
A_2	-0.25	0.25	-0.75	-0.25	0.00	0.00	0.00	0.00	0.00	0.375	-0.25	0.00

From the results of the Table 1, it has been observed that these existing score and accuracy functions can not rank to these numbers.

Example 2: If, we consider two IVIFNs $\mathrm{A}_1 = \langle[0.2, 0.6],[0.2, 0.4]\rangle$ and $\mathrm{A}_2 = \langle[0.3, 0.5],[0.1, 0.5]\rangle$ and applying the existing score and accuracy functions defined in Eqs. (3), (4),(5), (6), (7), (14), (16), (18), and (19), then obtained results are summarized in Table 2.

Table 2

	$S(.)$	$H(.)$	$M(.)$	$L(.)$	$T(.)$	$C(.)$	$S_{NWC}(.)$	$G(.)$	$S_{ivF}(.)$
A_1	0.1000	0.7000	0.1000	0.2400	0.2600	0.5500	-0.1400	0.6656	0.5000
A_2	0.1000	0.7000	0.1000	0.2400	0.2600	0.5500	-0.1400	0.6656	0.5000

From the results of the Table 2, it has been observed that these existing score and accuracy functions $S(.)$, $H(.)$, $M(.)$, $L(.)$, $T(.)$, $C(.)$, $S_{NWC}(.)$, $G(.)$ and $S_{ivF}(.)$ can not rank to these numbers.

Example 3: Let $\mathrm{A}_1 = \langle[0.3, 0.3],[0.3, 0.3]\rangle$ and $\mathrm{A}_2 = \langle[0.2, 0.2],[0.2, 0.2]\rangle$ two IVIFNs as alternatives. For ranking to these number, we utilized the existing ranking function defined in Eqs. (14), (15), (16) and (17) and obtained results are summarized in Table 3.

Table 3

	$C(.)$	$f(.)$	$S_{NWC}(.)$	$H_{NWC}(.)$
A_1	0.50	0.50	0.0	0.0
A_2	0.50	0.50	0.0	0.0

From the obtained results, it is clear that the score and accuracy functions $C(.)$, $f(.)$, $S_{NWC}(.)$, $H_{NWC}(.)$ can not rank the alternatives.

Example 4: Let two alternatives $A_1 = \langle [0.0, 0.1], [0.2, 0.3] \rangle$ and $A_2 = \langle [0.0, 0.5], [0.1, 0.6] \rangle$. To rank these alternatives, we apply the existing score and accuracy functions defined in Eqs. (10), and (12), and we get $P(A_1) = P(A_2) = 0.00$ and $J(A_1) = J(A_2) = 0.02$ which represents that functions $P(.)$, and $J(.)$ are fail to rank the alternatives.

From the result of the above examples, it is clear that the existing score and accuracy function have various shortcoming and rae unable to rank the numbers.

5. Conclusions

In this article, we review the existing score and accuracy functions defined by the authors Bai [3], Garg [6], Joshi and Kumar [7], Nayagam et al [8, 9], Priyadharsini and Balasubramaniam [11], Sahin [12], Wang and Chen [13, 14], Xu [15], Ye [16], Zhang and Xu [18], Zhang et al [19], and Nguyen [10], and some counter examples are presented to show the shortcomings of the existing score and accuracy functions. From the results of the counter examples, it has been concluded that the existing score and accuracy functions do not work perfectly and are unable to rank the IVIFNs. In future, this study will encourage to develop a new method for ranking the IVIFNs.

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